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Automatic Prior Design of LQG Adaptive Controllers

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Abstract

The results achieved in preparation of computer tools for the automatic commission of a multivariate LQG adaptive controller are reported, i.e. *unified theory* and a set of cooperating *algorithms* needed. The paper points out the general conception of the underlying practical project, refers to ready solutions of partial tasks and formulates the remaining problems to be solved.

1 Introduction

The advanced control theory is introduced into practice either in very demanding cases, where simple means have failed, or in controlling the processes, where any small performance improvement brings significant profit. In these cases much effort is devoted to study of the process and its possible modelling. The gathered information often cannot be fully exploited for the design due to lack of tools for including it into final controller specification.

One of the advanced controllers, now theoretically well developed for practical use, is the LQG multivariate adaptive controller, which approximately minimizes quadratic criterion using recursively estimated ARX linear model with Gaussian disturbances. For using fully its power and adaptivity, the controller needs careful specification of its optimal structure and working conditions under given technical constraints. The design often requires much knowledge, skill and experience.

The paper is devoted to discussion of computer aided design, where controller specification can proceed nearly automatically, regulated by user-oriented dialogue. Such a project proved to be quite feasible; its first version has been working for several years [10, 11, 12, 15]. Under the unifying viewpoint of probabilistic reasoning, more powerful version is being prepared for implementation.

Usual control design. In the adaptive context, the main stream of design procedures is oriented to an on-line exploitation of artificial intelligence (AI) tools as a second control layer. The prior design reduces mostly to off-line identification complemented typically by a robust control design. These parts are often combined by a human expert, sometimes by his AI substitute. This proceeding lacks unified approach as well as appropriate tools for exploiting existing prior information.

Aim of the paper. The very existence of the *algorithmic* solution of the prior design task for the adaptive controllers has not been appreciated yet. It has been opened by the algorithmic prediction of closed-loop properties, which is possible in the Bayesian set-up, rarely used in this area until now. We want to call attention to this research line, to refer to papers (not widely available) which offer solutions of some interesting partial problems and, first of all, to point out the challenging research problems.

Central problem. When applying control theory, the efficient use of system-oriented information is a weak point. This information consists of data measured on the controlled system, user's knowledge of the system, control aims, and restrictions. All this information should be used to design the controller, namely, to choose its structure, to initialize the estimator, to select the forgetting and penalizations.

Methodology. To seize the uncertainty of input information, the Bayesian approach is being used. All uncertain quantities are described by probability distributions. Thus, prior information merges with data by Bayes' rule and the parameter uncertainty is projected into the uncertainty of parameter-dependent variable by the substitution formula of probability calculus.

Key research issues. The extreme dimensionality of the computations and nonlinearity of the inspected transformations make the whole problem highly non-trivial. These obstacles can be handled by thoughtful modelling, tailored optimizations, Bayesian Monte Carlo methods and by the Bayesian treatment of decision subtasks.

2 Control design under uncertainty

This section serves for introducing notations and recalling the conceptual simplicity of the Bayesian design.

2.1 Theory

The designed controller influences observable data sequences $\mathcal{D}$ by optional inputs $\{u(\tau) \in u^*(\tau)\}_{\tau \in \tau^*}$.[1] An external (input-related) *scheduling* decides

[1] The symbol x^* means a set with a generic point x.

(through an increasing sequence of pointers $\tau \in \tau^*$) which input is to be designed. The input u decomposes $\mathcal{D}$ into the (input-related) *past* $\mathcal{P}_u$ and its *future* $\mathcal{F}_u$, i.e. $\mathcal{D} = (\mathcal{P}_u, u, \mathcal{F}_u)$. Consequently, (admissible) controllers are mappings $\{\mathcal{C}_\tau : \mathcal{P}^*_{u(\tau)} \to u^*(\tau)\}_{\tau\in\tau^*}$.

Hereafter, $\tau_{\mathcal{F}}$ denotes the nearest successor of a generic τ.

The system output $y(\tau)$ consists of the entries observed both in $\mathcal{F}_{u(\tau)}$ and in $\mathcal{P}_{u(\tau_{\mathcal{F}})}$. The (controlled) systems are formalized as uncertain (random and/or incompletely known) mappings $\{\mathcal{S}_\tau : \mathcal{P}^*_{u(\tau)}, u^*(\tau) \to y^*(\tau)\}_{\tau\in\tau^*}$.

The control quality is measured by a loss function $\mathcal{L}_\omega : \mathcal{D} \to [0,\infty]$ determined by a "shaping" parameter $\omega \in \omega^*$. The *optimal (admissible) controller* minimizes the expected loss $\mathcal{E}_{\mathcal{D}}[\mathcal{L}_\omega(\mathcal{D})|\text{prior knowledge}]$. Here, $\mathcal{E}_\bullet[\cdot|\circ]$ denotes the expectation with respect to $\bullet$ conditioned on $\circ$.

Proposition 1 [*Dynamic programming*] The optimal controller, if it exists, can be chosen as deterministic mappings of pasts to the related minimizing arguments in the functional equations
$\mathcal{V}(\mathcal{P}_{u(\tau)}) = \min_{u(\tau)\in u^*(\tau)} \mathcal{E}_{y(\tau)}[\mathcal{V}(\mathcal{P}_{u(\tau_{\mathcal{F}})})|\mathcal{P}_{u(\tau)}, u(\tau)], \quad \tau \in \tau^*$,
where the terminal $\mathcal{V}(\mathcal{D})$ equals to the loss function $\mathcal{L}_\omega(\mathcal{D})$.

$\square$

In a generic case, the design requires a *system model* in the form of the conditional probability density functions (p.d.f.) $\{p(y(\tau)|\mathcal{P}_{u(\tau)}, u(\tau))\}_{\tau\in\tau^*}$ (as usual, p.d.f.s $p(\cdot)$ are distinguished through the argument which may also mean the corresponding random variable or its realization).

Lack of complete knowledge, modelled by availability of a *parametrized system model* $\{p(y(\tau)|\mathcal{P}_{u(\tau)}, u(\tau), \Theta)\}_{\tau\in\tau^*}$ known up to an unknown parameter Θ, determines the scenario faced by adaptive controllers. This lack has to be counter-acted using prior knowledge and the observed data. A Bayesian view-point (*incompletely known* $\equiv$ *random*) leads to probability theory as the relevant formalism. A prior p.d.f. $p(\Theta)$ reflects the prior knowledge and the posterior p.d.f. $p(\Theta|\mathcal{P}_u)$ expresses the information after observing $\mathcal{P}_u$.

Proposition 2 [*Bayesian estimation and prediction*] For an admissible controller, it holds

$$p(y|\mathcal{P}_u, u) = \mathcal{E}_\Theta[p(y|\mathcal{P}_u, u, \Theta)|\mathcal{P}_u]. \tag{1}$$

The needed posterior p.d.f. is given by the Bayes rule

$$p(\Theta|\mathcal{P}_u) = \frac{L(\Theta, \mathcal{P}_u)}{\mathcal{I}(\mathcal{P}_u)} p(\Theta) \propto L(\Theta, \mathcal{P}_u) p(\Theta) \tag{2}$$

with the likelihood function $L(\Theta, \mathcal{P}_u) = \prod_{y(\tau),u(\tau)\in\mathcal{P}_u} p(y(\tau)|\mathcal{P}_{u(\tau)}, u(\tau), \Theta)$.
Furthermore,

$$\mathcal{I}(\mathcal{P}_u) = \mathcal{E}_\Theta[L(\Theta, \mathcal{P}_u)], \tag{3}$$

i.e. the prior expectation of the likelihood; $\propto$ means proportionality.

The system model (1) is also called predictive p.d.f. and has the alternative form $p(y(\tau)|\mathcal{P}_{u(\tau)}, u(\tau)) = \mathcal{I}(\mathcal{P}_{u(\tau_{\mathcal{F}})})/\mathcal{I}(\mathcal{P}_{u(\tau)})$.

$\square$

The estimation converges generically to a fixed point in Θ^*. In order to reach "adaptivity", changing properties of the controlled system have to be modelled as variations of the unknown Θ. The controller relies on implicit model of Θ-changes modelled by forgetting [4].

Proposition 3 [*Forgetting as Bayesian decision making*] Let the p.d.f.s which should be updated by $u(\tau), y(\tau)$ to $p(\Theta|\mathcal{P}_{u(\tau_{\mathcal{F}})})$ be restricted to the extreme pair:
- $p(\Theta|\mathcal{P}_{u(\tau)}) \Leftrightarrow$ the estimated parameter has not changed between τ and $\tau_{\mathcal{F}}$
- $p(\Theta) \Leftrightarrow$ Θ-variations reduce the guaranteed information to the prior one.

Let prior probabilities of both possibilities be $0 < \lambda \leq 1$, $1 - \lambda$, respectively. Then, the best p.d.f., minimizing its expected Kullback-Leibler distance to the unknown p.d.f., is proportional to $p^{\lambda}(\Theta|\mathcal{P}_{u(\tau)})p^{1-\lambda}(\Theta)$.

$\square$

2.2 The addressed problem

The design under uncertainty is fully described by Proposition 1; the needed estimation and prediction by Proposition 2 and the "tracking" extension by Proposition 3. At least since sixties [2], this elegant consistent design is known to suffer from the dimensionality curse. An LQ design *with given model* is a rare exception. This explains its wide use and popularity of its certainty-equivalence extension (Θ substituted by an estimate) in adaptive control. Fortunately, this extremely special case with the simple-minded sub-optimal strategy has been found successful in a wide range of difficult real life problems *if the free "user's knobs" were properly selected*; a complete computer support of their choice is, however, still missing.

Such a computer support is realizable, as outlined in the following. The possible solution is *analytical* (in the nomenclature MacFarlane et al. 1989), but its practical usefulness and efficiency still requires some theoretical and algorithmic preparations.

3 Free options of the LQG adaptive controller

Recalling briefly the LQG design, the formulation elements dictated by availability of the algorithmic solutions and the options left are pointed out.

3.1 Gaussian ARX model

The Gaussian ARX model, one of the few models admitting a finite-dimensional sufficient statistic, reads $p(y(\tau)|\mathcal{P}_{u(\tau)}, u(\tau), \Theta) = \mathcal{N}_{y(\tau)}(\theta'\psi(\tau), r)$, where $\mathcal{N}_y(\hat{y}, r)$ denotes the Gaussian p.d.f. of y specified by the mean $\hat{y}$ and covariance (matrix) r. The unknown parameter Θ consists of the regression coefficients θ and the covariance r ($'$ denotes transposition). Entries of the regression vector $\psi(\tau)$ contain $u(\tau)$ and a selection of several delayed data measured in its past $\mathcal{P}_{u(\tau)}$.

The model can be viewed as a Taylor-expansion based approximation of a general, sufficiently smooth, system behaviour. There is, however, always a danger that the approximation is too crude. A kind of pre-filtering can improve the situation.

Raw data pre-filters, structure of the regressor ψ and possibly time scheduling (sampling rate) are free options connected with this model.

3.2 Priors in estimation of ARX model

Proposition 4 [*Likelihood for ARX model*] For the ARX model, the likelihood $L(\Theta, \mathcal{P}_u) = L((\theta, r), \mathcal{P}_u)$ takes Gauss-inverse-Wishart (GiW) form

$$GiW_{\theta,r}(\nu, V) \propto |r|^{-0.5\nu} \exp\left\{-0.5\mathrm{tr}\left[r^{-1}\begin{bmatrix}-I\\ \theta\end{bmatrix}' V^{-1}\begin{bmatrix}-I\\ \theta\end{bmatrix}\right]\right\} \tag{4}$$

where the sufficient statistic is the positive scalar $\nu = \nu(\mathcal{P}_u)$ and positive semidefinite matrix $V = V(\mathcal{P}_u)\,(\geq 0)$ (I means the appropriate unit matrix). The transition $\mathcal{P}_{u(\tau)} \rightarrow \mathcal{P}_{u(\tau_\mathcal{F})}$ updates the statistics as follows

$$\nu(\tau_\mathcal{F}) = \nu(\tau) + 1, \quad V(\tau_\mathcal{F}) = V(\tau) + \Psi(\tau)\Psi'(\tau) \tag{5}$$

with $\Psi'(\tau) = [y'(\tau), \psi'(\tau)]$. The recursions start from zero values.

□

Proposition 5 [*Self-reproducing priors, Bayesian estimation*] The Bayesian estimation of the ARX model reduces to an algebraic recursion which determines a posterior p.d.f. with explicitly computable moments iff a self-reproducing prior $p(\theta, r) = GiW_{\theta,r}(\nu_0, V_0)$, $\nu_0 > m_\Psi$, $V_0 > 0$, is chosen (m_x denotes the dimension of a vector x).

The corresponding posterior p.d.f. is of the GiW-type (4). The recursions (5) are preserved with *zero initial conditions replaced by ν_0, V_0*. The recursion is equivalent to recursive least squares (RLS).

The normalizing factor has the structure

$$\mathcal{I}(\nu, V) = \Upsilon(\nu, m_\psi, |V|, |V_\psi|) \tag{6}$$

where the (known) function Υ depending on V and its submatrix V_ψ is updated solely by ψ.

$\square$

The quantities ν_0, V_0, or, equivalently, RLS initial conditions, are free options specifying the prior p.d.f.

3.3 Parameter tracking in ARX model

Proposition 6 [*Forgetting for ARX model*] For the ARX model and self-reproducing prior p.d.f., the forgetting (Proposition 3) preserves the GiW structure and modifies (5) to $\nu(\tau_{\mathcal{F}}) = \lambda\nu(\tau) + (1-\lambda)\nu_0 + 1, \;\; V(\tau_{\mathcal{F}}) = \lambda V(\tau) + (1-\lambda)V_0 + \Psi(\tau)\Psi'(\tau)$ (to weighted RLS).

$\square$

(Notice permanent influence of the prior information on tracking!)

The probability $\lambda, 0 < \lambda \leq 1$, known as forgetting factor, is the free option to control tracking abilities of the estimation.

3.4 LQ control design

Using certainty equivalence principle, the synthesis part coincides with the linear, known-parameter case. Consequently, the input range and the loss function form are decisive for realizability. For brevity, we discuss the regulation problem only, with the loss function

$$\mathcal{L}_\omega(\mathcal{D}) = \sum_{\tau \in \tau^*} \left(||y(\tau)||^2_{Q_y} + ||u(\tau)||^2_{Q_u} \right) + ||\psi_T||^2_{Q_T} \tag{7}$$

where $||x||^2_{Q_x} = x'Q'_xQ_xx$, the subscript $_T$ points to the last point in τ^*.

Proposition 7 [*Optimal LQ controller*] For the known-parameter ARX model, the quadratic loss (7) and unrestricted inputs, the optimal inputs are generated by the linear controller $C'_{opt,\tau}\psi(\tau) = 0$. The optimal control laws are specified by a simple function $\mathcal{G}_\omega$ of the regression coefficients θ and of a square (Riccati) matrix Q_τ ($C_{opt,\tau} = \mathcal{G}_\omega(\theta, Q_\tau)$). The matrix Q_τ evolves backwards, against the scheduling

$$Q_\tau = \mathcal{R}_\omega(\Theta, Q_{\tau_{\mathcal{F}}}), \tag{8}$$

starting from Q_T. The mapping $\mathcal{R}$ describes Riccati-equation equivalents.

$\square$

The available shaping knobs for expressing and combining control aims as well as influencing data ranges are $\omega = [Q_y, Q_u, Q_T]$.

4 User's formulation

The following outline of a user's problem formulation underlines substantial differences of his/her options (pieces-of-knowledge/wishes) from those of the LQG formulation.

4.1 Who is the user?

As underlined in the introduction, the complex adaptive controllers are used for difficult/complex controlled systems, for which successful implementation of any controller is complex, too, requiring a lot of knowledge about the process, technological restrictions and significance of various sub-goals. At the same time, deep knowledge of the adaptive controllers cannot be assumed at the user's side.

Thus, *wide knowledge of the controlled process and restricted knowledge of the LQG adaptive controller are expected from the user.*

4.2 User's knowledge

The knowledge of a typical user, for whom an algorithmic support is worth preparing, is not well mapped yet: *its specification is one of sub-problems of the research.* The knowledge items usually available are:

Data measured on the system and/or generated by a carefully built-up simulation model of a high complexity unsuitable for control design.

Prior knowledge consisting of expected (designed or recorded in past) properties like: gains, typical responses on typical stimuli (step responses, (parts of) frequency responses), time constants, ranges of signals and their changes, important frequency ranges, information about stability of various loops, levels and frequencies of measurement noises.

4.3 User's wishes and restrictions

A typical formulation could be reduced to the simple dictum:

> Keep important measured quantities as close as possible to a pre-specified ideal behavior without violating restrictions on other ones.

In the regulation problem, it may look as follows:

Keep a (controlled) part of the output as close to zero as possible while keeping input, input changes and some of the remaining outputs and their changes in prescribed intervals.

Usually, *the user expresses his/her wishes in "extremal" terms under hard restrictions without any explicit loss specified.*

5 Mapping user's options to LQG options: the research aim, state and directions

The research aims at creating a Bayes-based tool set for a computerized "translation" of the free user's options to the free options of the adaptive LQG controller.

The discussion below outlines the state of the translation art and the research directions in solving open problems.

5.1 Signal pre-filtering

State of art: A lot of (pre-)filters is ready if the noise is well separated from the signal. Difficulties are faced when outliers passing obvious range-tests, irregularities in the signal sampling or a similarity of the noise to the signal are encountered. Local filters proposed and successfully tested in past seem to be a common cure. They are based on the Bayesian estimation of local parametrized simple models of the useful signal $s(t)$ with (partially) inherited prior p.d.f.s. The regression line $p(s(t)|\mathcal{P}_{u(\tau)}) = \mathcal{N}_{s(t)}(\theta_1(\tau) + \theta_2(\tau)t, r(\tau))$ with $\Theta(\tau)$ constant for $t \in [\tau, \tau_{\mathcal{F}})$ is a typical example.

Research directions: Theory of local filters is unfinished yet and problem of fitting to the user's knowledge is unsolved. Preliminary reasoning leads to the conjecture that modelling of $s(t)$ by several-times integrated Wiener noise and measurements as its noisy samples will lead to a version of Kalman filter in Gaussian (no-outlier) case and to Bayesian non-linear but feasible (due to the model simplicity) estimation. Existing relations to a control period estimation (see below) should be studied, too.

5.2 Prior information

State of art: Off line estimation as well as merging expert knowledge in artificial intelligence (AI) are well developed arts. The joint use of all informational sources for the adaptive control has been, however, neglected outside Prague. At the same time, it deeply influences the reachable precision of the off line estimation and transients of its on-line mode with obvious (often rather strong) consequences on the reachable quality of robust and/or adaptive controller exploiting them.

The used methodology expresses any piece of information through fictitious data $\Psi'_f = [y'_f, \psi'_f]$ which could be observed on the system and treats them as if they were observed. For instance, if the static gain g of a SISO system lies with some probability in $[\hat{g} - \sqrt{r_g}, \hat{g} + \sqrt{r_g}]$, the fictitious data vector should be $\Psi'_f = [g \text{ at } y \text{ positions}, 1 \text{ at } u \text{ positions}]\sqrt{\hat{r}/r_g}$, ($\hat{r}$ is a prior guess of r).

The fictitious data suit the common quantification of all information sources (expert knowledge, preliminary data, analytical bounds, knowledge on frequency characteristics, simulation results etc.) including their uncertainty. Merging of particular items is, however, a non-trivial problem and mechanical use of Bayes rule is impossible.

Proposition 8 [*Conservative merging of fictitious data*] Let V_f be a sum of fictitious data dyads $\Psi_f \Psi'_f$ added to a matrix $V_{00} > 0$ (cf. (5)), expressing vague knowledge. Let us normalize V_f to V_0 so that the corresponding LS estimate of r coincides with its prior guess $\hat{r}$. Then, $p(\Theta) = GiW_{\theta,r}(\nu_0 = m_\Psi, V_0)$ is the best merger: the joint p.d.f. of fictitious innovations is the nearest to the p.d.f. of the ideal white-noise innovation sequence.

□

Research directions: The presented result is quite strong and offers plausible results but it is too much ARX-model dependent and its extensions are unclear. Moreover, careless treatment of informational groups of different cardinality might lead to unbalanced exploitation of the information at hand. It indicates need for a deeper understanding. A search for individual predictors based on the constructed prior p.d.f. with sufficient confidence assigned to expected (fictitious) output ranges seems to be promising.

Of course, a construction of Ψ_fs reflecting the core of user's knowledge has to be elaborated in detail. It requires learning this core and preparing the relevant fictitious data. Use of the prior p.d.f. in structure estimation enforces formulation of the prior knowledge in a global, model-structure independent way. For instance, stability should be formulated as bounded-input bounded-output type (exploiting the expected signal ranges) not as a location of transfer function poles.

5.3 Structure estimation

State of art: Structure estimation helps selecting "the best" of competitive parametrized models $\{p(y(\tau)|\mathcal{P}_{u(\tau)}, u(\tau), \Theta^i)\}_{\tau \in \tau^*}$, $i = 1, 2, \ldots, N < \infty$. The competitors describe relations among the same data (each of them may contain an individual unknown parameter Θ^i). In the Bayesian set-up, the description of our uncertainty on the best-model pointer $s \in \{1, 2, \ldots, N\}$ reduces to evaluation of $p(s|\text{available information})$. For its simple construction, the following "compound model" parametrized by $\Theta = (s, \Theta^1, \ldots, \Theta^N)$ is useful $\{p(y(\tau)|\mathcal{P}_{u(\tau)}, u(\tau), \Theta) = \prod_{i=1}^{N} p^{\delta_{is}}(y(\tau)|\mathcal{P}_{u(\tau)}, u(\tau), \Theta^i)\}_{\tau \in \tau^*}$. with δ_{is} denoting Kronecker symbol. Prior independence of the above product constituents seems to be a natural assumption giving the prior p.d.f. $p(\Theta) = p(s) \prod_{i=1}^{N} p(\Theta^i)$.

Proposition 9 [*Structure estimation*] Under the prior independence, it holds

$$p(s|\mathcal{P}_u) \propto \frac{\mathcal{I}^s(\mathcal{P}_u)}{\mathcal{I}^s(\mathcal{P}_0)} p(s) \tag{9}$$

where $\mathcal{I}^s(\mathcal{P}_u)$ is given by (3) for s-th structure and $\mathcal{I}^s(\mathcal{P}_0)$ is the normalizing factor of the related prior p.d.f. $p(\Theta^s)$, i.e. predictive p.d.f.s are constructed for all competitive models and the Bayes formula is applied taking the pointer s as the only unknown parameter left.

□

Research directions: Three sub-groups discussed below have their own research sub-problems. Their combination, however, is the most challenging problem.

5.3.1 Regressor structure

State of art: Decisions on model orders, time delays, significance of external measurable variables and influence of various inputs can be formulated as a choice of the regressor structure. A careful choice of the regressor structure avoids risks of under-parametrization or an increase of uncertainty due to over-parametrization. For the ARX model, the needed $\mathcal{I}^s$ are given by the explicit formula (6). Exponential burst of the number of the competitive structures makes the application non-trivial. A special hill-climbing algorithm was designed and successfully tested searching for the maximum of (9) with a vague prior p.d.f.'s $p(\Theta^s)$.

Research directions: The experimentally verified strong influence of the priors $p(\Theta^s)$ on the structure estimation motivates including them into the structure estimation.

The impossibility to recognize de-coupled sub-systems is the notable drawback of the used parametrization. The remedy is implied by:

Proposition 10 [*Single-output parametrization*] Let y_i, $i = 1, \ldots, m_y$ denote output entries. Any multivariate parametrized model of the type used can be rewritten $p(y|\mathcal{P}_u, u, \Theta) = \prod_{i=1}^{m_y} p(y_i|y_1, \ldots, y_{i-1}, \mathcal{P}_u, u, \Theta)$, Thus, any parametrized model can be treated as a collection of single output models (each with its own regressor structure).

□

In the ARX case, Proposition 10 leads to a model with fully independent entries of the noise vector and with the regressors $\psi_i(\tau)$ (related to $y_i(\tau)$) extended by $y_j(\tau)$, $j < i$. The sub-system decoupling is then recognized if $\Psi_i(\tau)$ does not contain entries of $\Psi_j(\tau)$. Search for the best structure has to be adapted accordingly and estimation of the best ordering of output entries (which could be viewed as an output-related scheduling) performed.

Together with the outlined progress further improvements of the used hill climbing will be inspected: The needed evaluation (6) of determinants of, generally large, matrices V_ψ, V, is feasible only if their factorized versions (say, Choleski square roots G_ψ, G) are dealt with. The maximization procedure was motivated by need to make the transition $G^{s_{old}} \to G^{s_{new}}$ computationally cheap when comparing a new structure guess s_{new} with an old one s_{old}. If $\Psi^{s_{new}}$ is nested in $\Psi^{s_{old}}$, i.e. $\Psi^{s_{old}} = [\Psi^{s_{new}}, \ldots]$, then the square root $G^{s_{new}}$ is nested in $G^{s_{old}}$, too. This observation has not been exploited completely yet.

5.3.2 Scheduling - control period

State of art: As a rule, the input changes periodically with rate determined by the control period which is one of the most important characteristics of any digital controller. In model based controllers, the proper period balances a model predictive efficiency with mismodelling errors. A Bayesian solution makes this search systematic. It is based on structure estimation applied to models parametrized by control periods n which are integer multiples of a given sampling period.

Proposition 11 [*Estimation of control period*] Let the parametrized models of the single output model corresponding to a control period n be $\mathcal{N}_{y(\tau)}(\hat{y}(\tau), r)$ with $\hat{y}(\tau) = 1_n \theta' \psi(\tau)$, $r = R(I + \frac{\rho-1}{n} 1_n 1_n')$, $R, \rho > 0$, where 1_n is the n-vector of units and the delayed outputs in the regressor are compressed in sample means $y'(\bar{\tau}) 1_n / n$, $\bar{\tau} < \tau$. Then,
1. the posterior probability of sth periods is given by the formula (9) with a priori expected likelihood $\mathcal{I}^s(\mathcal{P}) = \bar{\Upsilon}(\nu, m_{s,\psi}, |V_s|, |V_{s,\psi}|, \Lambda_s)$. The function $\bar{\Upsilon}$ depends on statistics described in Proposition 5 (for the structure s) and a new statistic Λ_s measuring inter-sample deviations of the output entries $y(\tau)$ from their average $\bar{y}_n(\tau) = n^{-1} \sum_{i=1}^{n} y_i(\tau)$;
2. the parameter estimation reduces to that of the ARX model in $\bar{y}_n(\tau)$ if the control period n is applied.

□

Note that the model predicts the (vector) output to concentrate around its average.

Research directions: The solution has to be extended to multivariate case. Application to the parametrization implied by Proposition 10 seems to be straightforward but multi-rate case is expected to be non-trivial due to the evaluation complexity. Experience gained in the regressor structure estimation might be applicable.

Full exploitation of the general notion of scheduling – any multi-input case can be viewed as a set of single-input ones if scheduling runs over time and vector-input entries – could lead to substantially deeper insight into the controller structure design. For instance, proper choice of tightly connected input-output pairs could be recommended by structure estimation. This problem should be studied but definite results are not guaranteed.

5.3.3 Forgetting

State of art: A methodology of constructing the prior statistics ν_0, V_0 has been described above. Thus, the choice of scalar forgetting factor (probability) $\lambda \in (0,1]$ has to be faced. A simple inspection of estimation results for several values of λ proved to be satisfactory.

Research directions: The inspection will be made automatic and rigorous by taking the discrete values of λ as structural pointers and using Bayesian structure estimation.

The value of the "best" forgetting can be viewed as a measure of model non-stationarity. Use of the forgetting in off-line identification opens a conceptual problem of the relevance of the corresponding predictions (a sort of stationarity on which it relies).

5.3.4 Combined structure estimation

State of art: The described key sub-directions of structure estimation are clearly interrelated and, at the end, they have to be combined. Almost nothing has been done in this respect.

Research directions: The combined estimation, especially simultaneous estimation of the regressor-structure and control-period, reveals the basic methodological problem of the Bayesian set-up which assumes (implicitly) that the system generating data is describable by one of the inspected models: $\Theta_{true} \in \Theta^*$. An approximation-theory interpretation of statistics [3] seems to be promising remedy. This could lead to guidelines how to combine the discussed particular solutions (possibly modified) into a complete one. Even if a rigorous solution will not be found a heuristic mixing can be applied (an iterative search for a compromise among structural aspects which can partially substitute each other).

5.4 Summary of estimation

The presented theory provides guidelines for specifying the initial parameter probability $p(\Theta|\nu_0, V_0) = p(\Theta|$ full information collected in the design stage) according to the following schedule:
– quantify prior information and merge it with measured data: obtain V_0^i, ν_0^i for all competitive structures;
– select (a priori) the most probable structure $\hat{s}$ and fix it for on-line use (this decision task is dictated by obvious pragmatic reasons, however, the formal predictive p.d.f. needed for the controller can be evaluated according to (1) with the compound Θ);
– take $V_0^{\hat{s}}, \nu_0^{\hat{s}}$ as the initial conditions ν_0, V_0 for on-line estimation and as forgetting specification.

Successful research in the directions described can provide complete tools for *mapping prior user's knowledge to a fixed structure and initial conditions for on line estimation* ν_0, V_0 *and to the forgetting scheme.*

5.5 Prediction of closed loop behavior

State of art: Shaping parameter $\omega \in \omega^*$ is the only free option left for reflecting user's wishes and restrictions. The choice has to be based on comparison of predicted and required closed loop performance. A user quantifies the performance by a vector h of performance indices. For brevity we restrict our explanation to the dominating cases for which h is a known deterministic function of the model parameter Θ and of the shaping parameter ω, i.e. there is a known function $H : (\omega^*, \Theta) \rightarrow h^*$.

Proposition 12 [*Closed-loop performance prediction* $\leftrightarrow$ *static design under uncertainty*] Let $\mathcal{L}_d : h^* \rightarrow [0, \infty]$ measure user's dissatisfaction. Then, the optimal ω minimizing the expected value of $\mathcal{L}_d$ is the minimizing argument in the expression $\min_{\omega\in\omega^*} \mathcal{E}_h[\mathcal{L}_d(h)|\omega, \nu_0, V_0]$. Thus, the full information needed for arbitrary $\mathcal{L}_d(h)$ is $p(h|\omega, \nu_0, V_0)$ corresponding to the distribution function $Probability(h \leq \bar{h}|\omega, \nu_0, V_0) = \mathcal{E}_\Theta[Indicator(\{H(\omega, \Theta) \leq \bar{h}\})|\nu_0, V_0]$

□

Proposition 12 shows that a deductive prior analysis of a design under uncertainty is possible. Standard design (a parameter estimate found using an identification package transferred into control design package, or identification - robust control) is an approximation of the above optimal procedure which puts unnecessary load on users and their skill. Clearly, feasibility of the $p(h|\omega, \nu_0, V_0)$ computation is decisive for the usefulness of this line. A Monte-Carlo evaluation seems to be the only, and previous attempts indicate realistic, candidate, as H is complex, non-linear mapping specified mostly numerically. The conceptual algorithm reads:

1. generate independent samples Θ_i having p.d.f. $p(\Theta|\nu_0, V_0)$;
2. evaluate (independent) samples $h_i = H(\omega, \Theta_i)$ for the inspected ω;
3. use h_i for estimating $p(h|\omega, \nu_0, V_0)$.

Each step of this simple procedure hides difficult problems to be solved. The key ones are:
– fast evaluation of h_i: the number of needed samples tends to be large;
– selection of the procedure for estimating infinite dimensional $p(h)$;
– definition of a well justified stopping rule.

Research directions: Possibilities of fast evaluations have not been fully exploited yet. Majority of usual quality measures, like stationary signal dispersions, probabilities with which stationary signal will stay in some range, probability of stability etc., are simple functions of the stationary state expectation and covariance. For instance,

$$P_u \equiv Probability(\text{stationary } u \in \text{ interval}) = \tilde{\Upsilon}(\bar{u}, r_u)$$

where the function $\tilde{\Upsilon}$ depends on stationary expectation and dispersion of u (for the given Θ and control law). The stationary moments can be found by using fast/parallel signal-processing and/or algebraic methods. Need for some extension of the indices of interests and related tailored evaluations is foreseen, e.g. overshoot is the often required index.

The decisive step toward practical feasibility was reached by parametrizing $p(h)$:

Proposition 13 [*Parametrized distributions of some quality measures*]
• $p(r_u) \approx$ lognormal
• $P_u \approx$ normal
• p(closed-loop stability) $\equiv$ Bernoulli distribution.

□

The above list contains examples of results found by experimental work, intuitive considerations and exact derivations. A systematic way of their derivation would be desirable. Even if this direction were not successful, the list has to be extended to other quantities like distribution of mutual correlations, overshoots, closed-loop poles dependent quantities, etc.

The parametrization decreases drastically the need for the amount of samples and makes possible to decrease it further through Bayesian sequential stopping rules like:

Proposition 14 [*Stopping based on P_u*] Let the loss resulting from generating K independent samples of P_u, say $P_u(1), \ldots, P_u(K)$, be $\sigma(K) + cK$

where $\sigma(K)$ is dispersion of P_u conditioned on the samples and $c > 0$ be a weight assigned to number of samples (measure of the computational load). Then, the optimal sequential stopping rule reads: stop if sample dispersion of LS estimate of $P_u \leq cK$ otherwise take a new sample.

$\square$

This Proposition has to be extended to other quantities or at least its multivariate version is needed.

5.6 Optimization of closed loop behavior

State of art: The accessibility of the closed-loop behavior prediction gives possibility to inspect different controllers (corresponding, for instance, to different options of ω). The conceptual algorithm looks as follows:
1. select trial shaping parameter ω^j;
2. generate independent samples Θ_i having p.d.f. $p(\Theta|\nu_0, V_0)$;
3. evaluate control law assigned to the chosen ω^j (using an equivalent to (8);
4. evaluate (independent) samples $h_i^j = H(\omega^j, \Theta_i)$ and return to generating Θ while estimate of $p(h|\omega^j, \nu_0, V_0)$ is not precise enough;
5. compare $p(h|\omega^j, \nu_0, V_0)$ with the user-specified desirable state: return to the first step while user is not satisfied.

The current solution inspects LQ control laws gained for the inspected ω and parameters coinciding with the sample of Θ, providing a reachable lower bound on the closed loop quantities:

Propositions 15 [*Quality of the optimally adapted LQ controller*] Let $\mathcal{C}^*$ be set of adaptive controllers and let $\mathcal{C}^*_\Theta$ be controllers designed for the known model parameter Θ. Then, $\mathcal{E}\left[\min_{\mathcal{C}^*_\Theta} \mathcal{E}[\mathcal{L}|\Theta, \nu_0, V_0]|\nu_0, V_0\right] \leq \min_{\mathcal{C}^*} \mathcal{E}[\mathcal{L}|\nu_0, V_0]$ with equality reached for the ideally adapted controller.

$\square$

The diagonal value of the weighting matrix Q_u in (7) is the only shaping parameter optimized by a simple golden-cross method which is efficient due to:

Propositions 16 [*Monotonic dependence of range probability P_r on Q_u*] For the diagonal Q_u, it holds $P_r = p(\text{stationary } u(\tau) \in \text{ interval})$ is increasing function of $Q_u' Q_u$.

$\square$

The inner loop in the above algorithm is stopped according to the Proposition 14 with the weight c decreasing with decreasing distance of a predicted and required decisive value of P_r (number of heavy evaluations again decreased).

Research directions: Many theoretical and algorithmic extensions are needed to make the above procedure useful for complex situations, e.g.:
– full multivariate solution is needed, respecting cross-terms in penalties under weaker versions of above monotonicity-type results which are expected to be found;
– fast/parallel equivalents of the mapping (8) have to be compared and the most suitable used;
– user's multi-criterial reasoning must be respected (at least by giving him a chance to interact with the optimization loop);
– comparison with other types of controllers is desirable.

As in prediction of closed loop behaviour, a lot of tailoring to the user's wishes has to be made in order to make all solution practically feasible. A general solution is obviously hopeless at least because of inherent infinite dimensionality of the problem (cf. need for $p(h|\cdot)$).

5.7 Summary of prior design

The estimation part will provide tools for specifying the prior parameter probability $p(\Theta|\nu_0, V_0) = p(\Theta|\,\text{full information collected in the design stage})$, i.e. *mapping prior user's knowledge to a fixed structure and initial conditions for on-line estimation of ν_0, V_0 and to the forgetting scheme.*

The p.d.f. $p(\Theta|\nu_0, V_0)$ compresses the information needed in the rest of the controller design. With it, the methodology will be completed *converting closed-loop prediction of user's vector criterion and restrictions into shaping parameter ω providing well grounded predictions and comparisons needed for deciding on economical impact of using the LQG adaptive controller.*

6 Conclusions

The design of a complex multivariate adaptive LQG controller is most successful when combining good knowledge of the process controlled with good knowledge of the control subtleties. The deepest information about the system can be expected at the user's; the control design can be substantially supported by computer programs exploiting as much user's prior knowledge as possible.

The paper describes theoretical grounds of such a support for the whole design procedure from the analysis of priors to the probabilistic evaluation of future results. The particular design problems are formulated, state of the art shortly summarized and open problems stated, possibly with foreseen solutions.

The paper can be taken either as a review of research results and a source of interesting practically motivated problems or as a challenging question to the designer of the advanced control systems:

Have you used the algorithmic optimization means available before taking recourse to the artificial intelligence tools?

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