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Supervisory Control of Parallelly Interconnected Timed Discrete Event Systems[1]

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Abstract— **The paper solves a control problem for a class of real-time discrete event systems. The problem can be viewed as a real-time extension of the Supervisory Control Problem introduced by P.J. Ramadge and W.M. Wonham. The considered plant is the parallel composition of subsystems with guaranteed least delays between two subsequent output discrete events and with true parallelism. The resulting controller is presented. Certain aspects of effectiveness are discussed.**

I. Introduction

Implementation of a supervisor designed within the approach of Ramadge and Wonham [1] but running in real-time is considered in [2, 3]. It is shown that this implementation does not satisfy in general the specifications of the original untimed control problem. Necessary and sufficient conditions for solvability of the timed extended problem are proved [2] for a class of real-time discrete event systems called the elementary discrete event systems (EDES). Any system of this class is characterized fully by means of a corresponding RW-System (a system of the class of logical discrete event systems introduced by Ramadge and Wonham [1]), time set, and a mapping describing least delays between two subsequent output discrete events [2] (see also Appendix A). This class is not closed with respect to parallel coupling of systems.

In the present paper, a plant composed of several parallel EDESs is considered. The control problem which can be viewed as a real-time extension of the Supervisory Control Problem (SCP) [1] is solved. Certain difficulties related to the tradeoff between the expressive power of control problem specifications and the effectiveness of computing are discussed (see Examples I.1 and I.2 for an informal explanation). These difficulties are not significant for a plant which consists of only single EDES. However, the difficulties seem to be typical for problems related to real-time discrete event systems with communication delays [4].

The contribution of the paper is:

1. Solution of a real-time control problem.
2. The solution gives also guidelines for safe real-time implementation of controller designed within the logical discrete event systems theory by Ramadge and Wonham for a class of timed DESs.
3. Discussion of certain difficulties that arises naturally in control problems related to real-time discrete event systems and suggestions how they can be overcome.

The paper is organized as follows. Examples I.1 and I.2 discuss informally the control problem and the mentioned difficulties. The plant is defined in Section III. The control problem is formulated and solved in Section IV. The results are discussed and future research is outlined in Section V.

Example I.1: Consider two subsystems $\mathcal{S}_1$ and $\mathcal{S}_2$ which run in parallel (with true parallelism) with untimed models [1] defined using sets of output event labels $\Sigma_1 = \{\mathrm{A}\}$, $\Sigma_2 = \{\mathrm{a}, \mathrm{b}\}$; languages generated by the systems $L_1 = \mathrm{A}^*$, $L_2 = \mathrm{a}^*\mathrm{b}$; sets of controllable events $\Sigma_{c,1} = \{\mathrm{A}\}$, $\Sigma_{c,2} = \{\mathrm{b}\}$, respectively, where "$\mathrm{a}^*$" denotes all finite sequences of elements "a" and "$\mathrm{a}^*\mathrm{b}$" is the concatenation. The occurrence of controllable events can be controlled by means of disablements or enablements. These commands form the input of the systems.

The systems run in real-time (time set is equal to non-negative reals $\mathbb{R}_0^+$). This is modelled by two EDESs [5, 3] $\mathcal{S}_1$ and $\mathcal{S}_2$. Suppose that generation of output discrete events is restricted by means of mappings $\varepsilon_1 : \Sigma_1^* \times \Sigma_1 \rightarrow \mathbb{R}_0^+$ and $\varepsilon_2 : \Sigma_2^* \times \Sigma_2 \rightarrow \mathbb{R}_0^+$, respectively describing the least delays between two subsequent output discrete events. The mapping ε_1 is identically equal to 10 and ε_2 is for all $w \in \Sigma_2^*$ and $\sigma \in \Sigma_2$ defined according to

$$\varepsilon_2(w, \sigma) = \begin{cases} 0 & \text{if for all } \sigma = \mathrm{b}, \\ 20 & \text{otherwise.} \end{cases}$$

The automata corresponding to the languages L_1 and L_2 are depicted on Figure 1, where also the least delays are symbolized using labeled lines inside of states.

If a least delay is equal to 0 then it means that there is no time restriction to the occurrence of output events. Note that it implies that the event can occur arbitrarily early after last output discrete event. It is supposed that for each closed interval $[0, t)$ for $t \in \mathbb{R}_0^+$ there is an (arbitrary) finite number of discrete event occurrences for each observation of a given EDES. The observation is supposed to be a trajectory of values observed on the output of the system. The least delay equal to zero can be interpreted either as no constraint as mentioned above or as no information about the delay (i.e. the worst case is considered). It is obvious that also non-zero least delays cannot be guaranteed in general if the events belong to different subsystems. This is the reason why this class of systems is not closed under the parallel composition.

Consider a language L_m recognized by the automaton on Figure 2 by the states q_1 and q_2. This language represents

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the marked language and it is equal to the legal language L_g in this case. The supervisory control problem is solvable as L_g is controllable.

It is shown [2] that certain events must be disabled "in advance" for EDESs if the controller is required to be strongly causal (i.e. not responding to current values on the input but to the past only). The analogy for parallelly interconnected EDES is Theorem IV.1. It means for the present example that the path "ab" must be considered for disablement at the state q_0, i.e. "b" must be disabled before "a" can occur. The reason is that there is the least delay equal to 0 between any event labeled by "a" and the event labeled by "b".

This means that after disablement "b" at q_0 the state q_2 becomes unreachable. It can be verified that "b" could be generated in the interval from 0 to 20, supposing that "b" is not disabled earlier than necessary. An important consequence is that the plant can generate more event sequences in general if they are generated as quickly as possible. If they are generated in certain "slow way" then some of them can become disabled. The computation of the intervals when events can be generated is possible in case of the plant formed by a single subsystem [2, 3]. But it becomes infeasible in general for the case of more subsystems as suggested in the following example. But note that the knowledge of these intervals is necessary for the proof of non-blocking and for satisfaction of the minimally acceptable behaviour. ■

It is shown in the present paper that it could be verified relatively effectively what events can occur in the case of slow generation events. Therefore the control problem is modified in the present paper such that for non-blocking and minimal acceptable behaviour only these events are considered. It makes the problem tractable and it also makes sense with respect to applications, as it gives possibility to specify which logical observations should be always possible to generate.

Example I.2: Consider the plant defined in the previous example and a language L_m recognized by the automaton on Figure 3 by the state q_e. The labels of simultaneous events are denoted by square brackets, e.g. "$\langle \mathrm{a}, \mathrm{A} \rangle$" denotes the label of a simultaneous event composed of two labels "a" and "A". The states named "OUT" on the figure denote some states that are not in $\mathrm{pref}(L_m)$. Recall that $\mathrm{pref}(L_m)$ is a set of all prefixes of words in L_m [1, 6, 5, 2]. These states should not be reached supposing that the SCP with L_m as the marked language and the legal language $L_g = L_m$ is considered. The reason is that the system would be blocked in these states, i.e. in our case uncapable of reaching q_e.

It can be observed that L_m is controllable with respect to the plant composed of two given RW-Systems, because all edges leading into the state "OUT" are controllable, i.e. they contain at least one controllable label. According to [1], this problem is solvable. The question is: "Is also the real-time problem (i.e. considering the given time set and the given least delays) solvable?"

Similarly as in previous example, the need for disablement of certain events in advance implies that e.g. the path "ab" must be considered for disablement at the state q_a, i.e. "b" must be disabled before "a" can occur. Similarly "b" must be disabled at the state q_b before "A" can occur. The reason is that there is the least delay equal to 0 between any event labeled by "a" and an event labeled by "b".

The only way how a word in L_m could be generated is such that an event labeled by "b" is generated immediately after at least three discrete events labeled by "A". The corresponding automaton recognizes this situation by the edge from q_a to q_e labeled by "b". After examination of the least delays, it can be shown that whenever the state q_a is entered, then it is possible for an event labeled by "a" to occur immediately. This implies that in order to avoid reaching the state q_d by means of control the event labeled by "b" would be disabled whenever the considered automaton is at the state q_a. But this implies (using the definition of the plant) that the state q_e cannot be ever reached. So the system is blocked. Therefore there is no solution of the considered real-time control problem. ■

Note that the decision about the possible occurrence of an event requires to examine not only parameters related to the paths leading to the states "OUT" but also paths that are recognized by another states. It is obvious that the solution of problems of this class (even if relatively simple) can have unacceptable complexity. The question is now: "What control problems can be solved effectively for the given class of plants?" The possibility to compute the control strategy on the basis of examination of paths recognized by the "OUT" states (not by another states) is regarded here as one of necessary requirements for the effectiveness of solution. Certain other aspects of effectiveness are mentioned in Section V.

The question is most likely impossible to be answered in a general way. But the problem can be reformulated such that the computations are much simpler but still the specifications are nontrivial and interesting with respect to some applications. The reason of these difficulties is that the controller must disable sometime in general some events after some time elapsed even if no events are generated in the meantime by the plant, because not only single events but whole paths which violates specifications of the control problem must be considered in general. This means that in general more paths can be generated if the output events are generated quickly then if they are generated slowly. But note that this "speed of generation" cannot be influenced by means of control for the EDESs. This effect was found already for plants formed by a single EDES [2], but an effective analysis was possible unlike in the case of parallel subsystems as shown in Example I.2. It is possible to decide effectively in our sense (i.e. using only information about paths leading out of the prefix and the delays related only to these paths but not on the basis of information how a particular state was reached) which events can occur and which not. This is not possible to do in general for plants composed of several EDESs. Similar difficulties arise even for a single EDES but with communications delays [4].

The way how these difficulties can be overcome is to make the specifications of the control problem independent of this "slow-way of output event generation". This makes sense also from the application point of view, because it is desirable to ensure that certain words can be generated by the closed-loop system under any circumstances, i.e. also in case of the "slow-way of output event generation". The modification of specifications affects the "non-blocking specification" and the specification related to the minimal acceptable behaviour.

II. Basic Concepts and Notation

The power set of a set B is denoted $\mathcal{P}(B)$, the empty set by $\emptyset$, and the set difference by $\setminus$.

For an arbitrary sets A and B, the set of all finitary ordinary mappings over A and B is denoted $\mathcal{F}_{\mathrm{fo}}(A, \mathcal{P}(B))$, and is defined as the set of all mappings f from A to $\mathcal{P}(B)$ such that the set $\{t \in T : f(t) \neq \emptyset\}$ is finite.

A finitary ordinary Timed Discrete Event System (TDES) is defined $\mathcal{S} = (T, \leq, U, \Sigma, \mathcal{G})$, where T, U and Y are sets, $\leq$ is a linear order in T, $U \cap Y = \emptyset$, and $\emptyset \neq \mathcal{G} \subseteq \mathcal{F}_{\mathrm{fo}}(T, \mathcal{P}(U \cup Y))$. The set T is the time set. The sets U and Σ are sets of input or output event labels, respectively. The set $\mathcal{G}$ is the set of input-output observations of the system, i.e. trajectories that can be observed on the system.

Composition of mappings f and g is defined if $\mathrm{Im}(g) \subseteq \mathrm{Dom}(f)$ such that for all $t \in \mathrm{Dom}(g)$, $(f \circ g)(t) = f(g(t))$. Domain (image) of a mapping f is denoted $\mathrm{Dom}(f)$ ($\mathrm{Im}(f)$), respectively. Restriction of a mapping f to a set B is denoted $f|_B$, i.e. $\mathrm{Dom}(f|_B) = \mathrm{Dom}(f) \cap B$. It is extended to sets of mappings

Let B be set. The masking m_V is defined as a class mapping on the universal class (class of all sets) such that it maps each set into its intersection with V. Example: $m_V \circ o$ denotes a mapping such that $\mathrm{Dom}(m_V \circ o) = \mathrm{Dom}(o)$ and for all $t \in \mathrm{Dom}(o)$ we have $(m_V \circ o)(t) = o(t) \cap V$.

Let $\mathcal{G}$ and $\mathcal{M}$ be sets of mappings. The set of all observations in $\mathcal{G}$ possible with respect to $\mathcal{M}$ is

$$\mathcal{O}_{\mathcal{G}}(\mathcal{M}) = \{g \in \mathcal{G} :\ \exists f \in \mathcal{M},\ g|_{\mathrm{Dom}(f)} = f\ \}.$$

The set of all observations in $\mathcal{G}$ possible with respect to $\mathcal{M}$ and the masking set V is

$$\mathcal{O}^V_{\mathcal{G}}(\mathcal{M}) = \{g \in \mathcal{G} :\ \exists f \in \mathcal{M},\ m_V \circ g|_{\mathrm{Dom}(f)} = f\ \}.$$

The set B is initial segment of an ordered set $(T, \leq)$ if $B \subseteq T$ and for all $b \in B$ and $t \in T$ such that $t \leq b$ we have $t \in B$. If in addition there exists $t \in T$ such that $b \leq t$ for all $b \in B$ then B is upper bounded initial segment of $(T, \leq)$. The set of all initial segments of $(T, \leq)$ is denoted by $\mathcal{I}_{(T,\leq)}$. The set of all upper bounded initial segments of $(T, \leq)$ by $\mathcal{I}^{\mathrm{b}}_{(T,\leq)}$.

Let the domain of each mapping $g \in \mathcal{G}$ be linearly ordered set for some order $\leq_g$. The set of all prefixes of $\mathcal{G}$:

$$\mathrm{pref}(\mathcal{G}) = \{g|_A :\ g \in \mathcal{G}\ \wedge\ A \in \mathcal{I}_{(\mathrm{Dom}(g), \leq_g)}\ \}\ .$$

The set of all upper bounded prefixes of $\mathcal{G}$:

$$\mathrm{prefb}(\mathcal{G}) = \{g|_A :\ g \in \mathcal{G}\ \wedge\ A \in \mathcal{I}^{\mathrm{b}}_{(\mathrm{Dom}(g), \leq_g)}\ \}\ .$$

The set of all continuations of $\mathcal{M}$ in $\mathcal{G}$:

$$\mathcal{C}_{\mathcal{G}}(\mathcal{M}) = \{g \in \mathrm{pref}(\mathcal{G}) :\ \exists f \in \mathcal{M},\ f \in \mathrm{pref}(g)\ \wedge\ f \neq g\}\ .$$

The set of all upper bounded continuations of $\mathcal{M}$ in $\mathcal{G}$:

$$\mathcal{C}^{\mathrm{b}}_{\mathcal{G}}(\mathcal{M}) = \{g \in \mathrm{prefb}(\mathcal{G}) :\ \exists f \in \mathcal{M},\ f \in \mathrm{pref}(g)\ \wedge\ f \neq g\}\ .$$

The empty (null) observation Λ_T is a mapping with its domain equal to T such that for all $t \in T$, $\Lambda_T(t) = \emptyset$.

The transformation of timed observations into the logical ones is defined as follows. For all $f \in \mathcal{F}_{\mathrm{fo}}(T, \mathcal{P}(V))$ we define an increasing sequence (of time moments of discrete event occurrences) $\hat{s}(f)$ such that $\mathrm{Im}(\hat{s}(f)) = \{t \in T : f(t) \neq \emptyset\}$.

For all $f \in \mathcal{F}_{\mathrm{fo}}(T, \mathcal{P}(V))$ we define a sequence $s(f)$ (of discrete event labels written in the order of their occurrence) such that $\mathrm{Dom}(s(f)) = \mathrm{Dom}(\hat{s}(f))$ and for all $i \in \mathrm{Dom}(s(f))$ we have $(s(f))(i) = f((\hat{s}(f))(i))$.

The transformations are extended to sets of observations such that $\hat{s}(\mathcal{G}) = \{\hat{s}(g) : g \in \mathcal{G}\}$ and $s(\mathcal{G}) = \{s(g) : g \in \mathcal{G}\}$.

Let there exists $\min T$. For an arbitrary $g \in \mathrm{prefb}(\mathcal{G})$, the time moment of the last discrete event in g is defined as follows

$$\mathrm{last}(g) = \max\{t \in \mathrm{Dom}(g) : g(t) \neq \emptyset \vee t = \min T\}\ .$$

The empty word is denoted by ϵ. The set of all finite sequences over a finite set Σ is denoted by Σ^*. The set of all finite sequences of non-zero length is denoted Σ^+. The number of occurrences of symbol σ in a finite sequence w is denoted by $|w|_\sigma$.

Closed and open intervals of real numbers are denoted using square brackets and parentheses, respectively, as usual. If the meaning is clear from the context then a set of natural numbers greater or equal to i and less or equal to j for some natural numbers i and j is denoted by $[i, j]$.

III. Parallelly Interconnected Elementary Discrete Event Systems

Consider some natural number p ($p \neq 0$) and RW–Systems $\mathcal{S}^{\mathrm{RW}}_i = (\Sigma_i, L_i, \Sigma_{c,i})$, $i \in [1, p]$, where the alphabets Σ_i are pairwise disjoint. If a RW–System runs in time and its timed features are modelled by a mapping describing the least delays between two subsequent output discrete events then a system of a class of elementary discrete event systems (EDES) is obtained [2, 3, 7] (the definition is summarized in Appendix A). The plant and the control problem can be defined and solved for different time sets. In order to avoid definitions necessary for dealing with general time sets [2, 6, 4], the paper is focused on one of the most interesting cases, namely non–negative real numbers $\mathbb{R}^+_0$. Let $\leq$ be the canonical order of reals.

Let for each $i \in [1, p]$ $\mathcal{S}_i = (\mathbb{R}^+_0, \leq, U_i, \Sigma_i, \mathcal{G}_i)$ be an EDES defined using the RW–System $\mathcal{S}^{\mathrm{RW}}_i$, the least delay mapping ε_i, a delay mapping E_i, a disablement mapping η_i, and an output law μ_i.

Consider a plant $\mathcal{S}$ formed by p parallelly interconnected EDESs $\mathcal{S}_i$, $i \in [1, p]$, defined above. Let from now on

$$U = \bigcup_{i \in [1,p]} U_i\ \ ,\quad \Sigma = \bigcup_{i \in [1,p]} \Sigma_i\ \ ,$$

$\mathcal{M} = \mathcal{F}_{\mathrm{fo}}(\mathbb{R}^+_0, \mathcal{P}(U \cup \Sigma))$, and for all $i \in [1, p]$ there is $V_i = U_i \cup \Sigma_i$.

The plant is modelled by a TDES $\mathcal{S} = (\mathbb{R}^+_0, \leq, U, \Sigma, \mathcal{G})$, where

$$\mathcal{G} = \{f \in \mathcal{M} :\ \forall t \in \mathbb{R}^+_0,\ m_\Sigma \circ f(t) \in \mu_o(f, t)\ \}\ ,$$

and $\mu_o : \mathrm{pref}(\mathcal{M}) \times \mathbb{R}_0^+ \to \mathcal{P}(\mathcal{P}(\Sigma))$ is the output law defined for all $f \in \mathrm{pref}(\mathcal{M})$ and $t \in \mathrm{Dom}(f)$ such that

$$\mu_o(f,t) = \{X_1 \cup X_2 \cup \ldots \cup X_p : \forall i \in [1,p],\ X_i \in \mu_i(m_{V_i} \circ f, t)\}.$$

Note that for all $f \in \mathrm{pref}(\mathcal{M})$ and $t \in \mathrm{Dom}(f)$ holds that $\emptyset \in \mu_o(f,t)$.

Consider from now on the plant $\mathcal{S} = (\mathbb{R}_0^+, \leq, U, \Sigma, \mathcal{G})$ defined above together with all its subsystems $\mathcal{S}_i$ and corresponding mappings ε_i, E_i, η_i.

The plant preserves the characteristic feature of EDESs, namely that the plant can always generate no discrete event in future (mapping Λ) independently on the past observation and on the input trajectory. This corresponds to the permissive control [1].

Let from now on

$$\mathcal{E} = \{e \in \mathcal{P}(\Sigma) \setminus \{\emptyset\} : \forall i \in [1,p], \mathrm{card}(\Sigma_i \cap e) \leq 1\} \quad (1)$$
$$\mathcal{E}_c = \{e \in \mathcal{E} : \exists i \in [1,p],\ \Sigma_{c,i} \cap e \neq \emptyset\} \ , \quad (2)$$
$$L = \{w \in \mathcal{E}^* : \forall i \in [1,p],\ s(m_{\Sigma_i} \circ w) \in L_i\} \ , \quad (3)$$

where "card" denotes cardinality of a set. The set $\mathcal{E}$ consists of all possible output discrete event labels except the null (empty) label. It includes all simultaneous labels (i.e. consisting of several labels). The set $\mathcal{E}_c$ forms all controllable events as it contains all labels e in $\mathcal{E}$ having at least one label $\sigma \in e$ in $\Sigma_{c,i}$ for some $i \in [1,p]$.

The language L is the model of output behaviour of the parallel interconnection of the RW–Systems with the true parallelism (i.e. the simultaneous events are considered). Note that each EDES produces on its output only single events not the simultaneous ones. But the parallel interconnection of several EDESs generates also simultaneous events.

Analogously as for EDESs, it holds that

$$\mathrm{pref}(s(m_\Sigma \circ \mathcal{G})) = s(m_\Sigma \circ \mathcal{G}) = L \ .$$

IV. Modified Timed Supervisory Control Problem

Let from now on $\mathcal{Z} \subseteq \mathcal{G}$. The set of logical output observations that can be "always" observed after f in $\mathcal{Z}$ is defined for all $f \in \mathrm{prefb}(\mathcal{Z})$ according to

$$\Pi_{\mathcal{Z}}(f) = \{v \in (\mathcal{P}(\Sigma) \setminus \{\emptyset\})^* : \forall g \in \mathcal{C}^{\mathrm{b}}_{\mathcal{Z}}(f),\ s(m_\Sigma \circ g) \in \mathrm{pref}(v)$$
$$\Rightarrow v \in s(m_\Sigma \circ \mathcal{O}_{\mathcal{Z}}(g))\ \}.$$

The set $\Pi_{\mathcal{Z}}(f)$ contains all output logical observations that can be observed in the future after f unless an output event violating them is generated. Note the following property: for all $f, g \in \mathrm{pref}(\mathcal{Z})$

$$g \in \mathrm{pref}(f) \Rightarrow \Pi_{\mathcal{Z}}(f) \subseteq \Pi_{\mathcal{Z}}(g) \ .$$

Therefore $\Pi_{\mathcal{Z}}(\Lambda_\emptyset)$ is the largest one.

The Modified Timed Supervisory Control Problem (MTSCP) is formulated as follows [4].

Control Problem IV.1: Let $\emptyset \neq L_a \subseteq L_g \subseteq L_m \subseteq L$ be parameters of the control problem.

Find a strongly controllable behaviour $\mathcal{Z}$ of $\mathcal{S}$ such that the following three specifications are satisfied

$$s(m_\Sigma \circ \mathcal{Z}) \cap L_m \subseteq L_g \ , \quad (4)$$
$$\forall f \in \mathrm{prefb}(\mathcal{Z}), \exists v \in \Pi_{\mathcal{Z}}(f), \quad v \in L_m \ , \quad (5)$$
$$L_a \subseteq \Pi_{\mathcal{Z}}(\Lambda_\emptyset) \cap L_m \ . \quad (6)$$

In [8], it is shown that for a set $\mathcal{Z}$ the fact that $\mathcal{Z}$ is strongly controllable behaviour is necessary and sufficient for existence of strongly causal controller which realizes the closed loop behaviour equal to $\mathcal{Z}$. The construction of controller is given in [8]. The definition of the strongly controllable behaviour is not presented here as it is not used directly and only certain results (Appendix B) related to it are necessary in the proofs.

Specifications (4) and (6) correspond to the original specification of the supervisory control problem [1]. Specification (5) expresses non-blocking property.

A. Necessary Conditions

In the rest of the paper, it is shown that the control strategy suggested by the following theorem is sufficient and moreover the minimally restrictive one.

Theorem IV.1: Let $s(m_\Sigma \circ \mathcal{Z}) \cap L_m \subseteq K \subseteq L$ for some K. If $\mathcal{Z}$ is a solution of Control Problem IV.1 related to the given plant then for all $f \in \mathcal{Z}$, $t \in \mathbb{R}_0^+$, and $w \in \mathcal{E}^+$ it holds

$$(\,(8) \wedge (9) \wedge (10)\,) \Rightarrow \quad (7)$$
$$(\exists i \in [1,p], \exists \sigma \in \Sigma_{c,i}, |z^i|_\sigma \neq 0 \wedge \sigma \in \eta_i(m_{V_i} \circ f|_{[0,t]}))$$

where $z^i = s(m_{\Sigma_i} \circ w)$, $z^i = \epsilon z_1^i z_2^i \ldots z_{n_i}^i$ for some natural number n_i, $v = s(m_\Sigma \circ f|_{[0,t)})$, $v^i = s(m_{\Sigma_i} \circ v)$, and

$$\forall i \in [1,p], (n_i \leq 1 \vee \forall j \in [2,n_i], \varepsilon_i(v^i z_1^i z_2^i \ldots z_{j-1}^i, z_j^i) = 0) \quad (8)$$
$$vw \in L \setminus \mathrm{pref}(K) \quad (9)$$
$$\forall i \in [1,p], \forall \tau \in \mathbb{R}_0^+ \setminus [0,t], (n_i \neq 0 \Rightarrow z_1^i \in E_i(m_{V_i} \circ f|_{[0,t)}, \tau)). \quad (10)$$

■

The following results present properties of the language $K = s(m_\Sigma \circ \mathcal{Z}) \cap L_m$, where $\mathcal{Z}$ is some solution of Control Problem IV.1. The following lemma proves that any path consisting of output event labels which form an event in $\mathrm{pref}(K)$ is also in $\mathrm{pref}(K)$.

The following mapping is necessary for the non-blocking condition as it represents all event labels that can be enabled relatively easily by means of control after observing v. Let $\emptyset \neq K \subseteq L_m$. We define $\beta : \mathrm{pref}(K) \to \mathcal{P}(\mathcal{E})$ such that for all $v \in \mathrm{pref}(K)$ we have

$$\beta(v) = \{e \in \mathcal{E} : ve \in L \ \wedge \ \forall w \in \mathcal{E}^+,\ (\,(8) \wedge (9)\,) \Rightarrow \quad (11)$$
$$(\exists i \in [1,p], \exists a \in \Sigma_{c,i},\ a \notin e \ \wedge \ |z^i|_a \neq 0)\} \ ,$$

$w = w_1 w_2 \ldots w_n$ for some natural number n, $z^i = s(m_{\Sigma_i} \circ w)$, $z^i = \epsilon z_1^i z_2^i \ldots z_{n_i}^i$ for some natural number n_i, $v^i = s(m_{\Sigma_i} \circ v)$.

The following is the key to the proof of non-blocking.

Lemma IV.1: Let $s(m_\Sigma \circ \mathcal{Z}) \cap L_m \subseteq K$. If $\mathcal{Z}$ is a solution of Control Problem IV.1 related to the given plant then for all $f \in \mathrm{prefb}(\mathcal{Z})$, $x \in \mathcal{E}^*$

$$\exists j \in [1,m],\ x_j \notin \beta(zx_1x_2\ldots x_{j-1}) \ \Rightarrow\ zx \notin \Pi_{\mathcal{Z}}(f) \ ,$$

where $z = s(m_\Sigma \circ f)$ and $x = x_1x_2\ldots x_m$ for some natural number m. ■

The following theorem summarizes the properties required from the language K.

Theorem IV.2: Let $s(m_\Sigma \circ \mathcal{Z}) \cap L_m = K$. If $\mathcal{Z}$ is a solution of Control Problem IV.1 related to the given plant then the following conditions are satisfied:

(i) $K \subseteq L_g$, $\mathrm{pref}(K) \cap L_m = K$, and K is a controllable language with respect to $\mathcal{S}^{RW} = (\mathcal{E}, L, \mathcal{E}_c)$,

(ii) $\forall x \in \mathrm{pref}(K), \forall e \in \mathcal{E}, \forall w \in \mathcal{E}^*$, $(xe \in \mathrm{pref}(K) \wedge \forall i \in [1,p], s(m_{\Sigma_i} \circ w) \in \{\epsilon, (e \cap \Sigma_i)\}\ ) \Rightarrow xw \in \mathrm{pref}(K)$.

(iii) $\forall v \in \mathrm{pref}(K), \exists w \in \mathcal{E}^*, (vw \in K \ \wedge\ \forall j \in [1,n],\ w_j \in \beta(vw_1w_2\ldots w_{j-1})\ )$,

(iv) $\forall w \in L_a, \forall j \in [1,n],\ w_j \in \beta(\epsilon w_1w_2\ldots w_{j-1})$,

where $w = w_1w_2\ldots w_n$ for some natural number n. ■

The first property can be expected from any real-time extension of the model and problem [1].

B. Sufficient Conditions

Lemma IV.2: Let $K \subseteq \mathcal{E}^*$ be a language controllable with respect to $(\mathcal{E}, L, \mathcal{E}_c)$. Let $\mathcal{Z}$ be a strongly controllable behaviour of the given plant. If

$$\forall f \in \mathcal{Z}, \forall t \in \mathbb{R}_0^+, \forall w \in \mathcal{E}^+, \quad (7) \qquad (12)$$

then $s(m_\Sigma \circ \mathcal{Z}) \subseteq \mathrm{pref}(K)$. ■

The previous lemma shows how it can be ensured that the logical output behaviour of the solution $s(m_\Sigma \circ \mathcal{Z})$ is subset of some language K. Theorem IV.2 gives certain properties of $s(m_\Sigma \circ \mathcal{Z}) \cap L_m$. But Example I.2 shows that it is not possible in general to compute effectively $s(m_\Sigma \circ \mathcal{Z})$. The following shows that a solution can be found even if $s(m_\Sigma \circ \mathcal{Z})$ is never computed. It is required only to compute a language $K \subseteq \mathcal{E}^*$ satisfying the four conditions of Theorem IV.2. This computation may be regarded relatively effective in comparison to the computation of $s(m_\Sigma \circ \mathcal{Z})$ as this requires to consider only paths with zero delays or path composed of labels belonging to different subsystems that lead out of $\mathrm{pref}(K)$.

Theorem IV.3: Let $K \subseteq \mathcal{E}^*$. If K satisfies the conditions (i) – (iv) of Theorem IV.2 then there exists a solution of the MTSCP such that $s(m_\Sigma \circ \mathcal{Z}) \cap L_m \subseteq K$. ■

A possible solution can be defined as follows. Let $\mu_c : \mathcal{G} \times \mathbb{R}_0^+ \to \mathcal{P}(\mathcal{P}(U)) \setminus \{\emptyset\}$ be a strong control law such that for all $f \in \mathcal{G}$ and $t \in \mathbb{R}_0^+$

$$\mu_c(f,t) = \{x \in \mathcal{P}(U) : \forall w \in \mathcal{E}^+, (\,(8) \wedge (9) \wedge (10)\,) \ \Rightarrow$$
$$(\ \exists i \in [1,p], \exists \sigma \in \Sigma_{c,i},\ |z^i|_\sigma \neq 0 \ \wedge\ \sigma \in \eta_i(m_{V_i} \circ u))\ \} \ ,$$

where $u \in \mathrm{pref}(m_U \circ \mathcal{G})$ such that $\mathrm{Dom}(u) = [0,t]$, $u|_{[0,t)} = m_U \circ f|_{[0,t)}$ and $u(t) = x$.

Let $\mathcal{Z} = \{f \in \mathcal{G} :\ \forall t \in \mathbb{R}_0^+,\ m_U \circ f(t) \in \mu_c(f,t)\ \}$.

Theorem IV.4: The set $\mathcal{Z}$ is a minimally restrictive solution of the Control Problem IV.1 related to the given plant such that $s(m_\Sigma \circ \mathcal{Z}) \cap L_m \subseteq K$. ■

V. Conclusion

The paper solves a real-time extension of the problem [1] for parallelly interconnected elementary discrete event systems. If the mappings ε_i are related in certain sense to the automata specifying languages L_i then the problem is computationally feasible. The requirements of Theorem IV.2 can be computed effectively in this case. Nevertheless, development of certain modular, decentralized, or hierarchical technique is necessary. The results [9, 10, 11, 12] can be used as guidelines for development of such technique.

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Appendix

A. Elementary Discrete Event Systems

The class of EDESs is defined for a general time set [2, 4]. In the present paper we consider the time set equal to $\mathbb{R}_0^+$ (non–negative real numbers). Let $\leq$ be the canonical order of reals and "$-$" be the subtraction.

This section gives the definition of an EDES indexed by $i \in [1, p]$ which is the subsystem of the plant considered in the present paper. Consider the RW-System $\mathcal{S}_i^{RW} = (\Sigma_i, L_i, \Sigma_{c,i})$ and a mapping $\varepsilon_i : \Sigma_i^* \times \Sigma_i \rightarrow \mathbb{R}_0^+$ describing the least delays.

Let $U_i = \{\mathbf{E}_\sigma : \sigma \in \Sigma_{c,i}\} \cup \{\mathbf{D}_\sigma : \sigma \in \Sigma_{c,i}\}$. The interpretation of the event labels $\mathbf{D}_\sigma$ is disablement of events labeled by σ, analogously $\mathbf{E}_\sigma$ is enablement. Let $\mathcal{M}_i = \mathcal{F}_{\text{fo}}(\mathbb{R}_0^+, \mathcal{P}(U_i \cup \Sigma_i))$.

A **disablement mapping** $\eta_i : \text{pref}(\mathcal{M}_i) \rightarrow \mathcal{P}(\Sigma_i)$ determining the set of disabled events is defined for all $f \in \text{pref}(\mathcal{M}_i)$

$$\eta_i(f) = \{\sigma \in \Sigma_i : \ \mathbf{E}_\sigma \in f(\text{last}(m_A \circ f))\ \} \quad ,$$

where $A = \{\mathbf{E}_\sigma, \mathbf{D}_\sigma\}$. It means that an event is disabled only if the disablement of σ was among the last control actions related to σ.

A **delay mapping** $E_i : \text{pref}(\mathcal{M}_i) \times \mathbb{R}_0^+ \rightarrow \mathcal{P}(\Sigma_i)$ describing the set of output event labels that are enabled with respect to the least delays at time t after observing f is defined for all $f \in \text{pref}(\mathcal{M}_i)$ and $t \in \mathbb{R}_0^+$ according to

$$E_i(f,t) = \{\sigma \in \Sigma_i : \ \varepsilon_i(s(m_{\Sigma_i} \circ f), \sigma) < t - \text{last}(m_{\Sigma_i} \circ f)\ \} \quad .$$

The model of Elementary Discrete Event Systems (EDES) has two components. The first is the set $\mathcal{M}_i$. The second component is an output law $\mu_i : \text{pref}(\mathcal{M}_i) \times \mathbb{R}_0^+ \rightarrow \mathcal{P}(\mathcal{P}(\Sigma_i))$ which gives for each $f \in \text{pref}(\mathcal{M}_i)$ and $t \in \text{Dom}(f)$ the set of output event labels that can be observed at the time t after observing $f|_{[0,t)}$ according to

$$\begin{aligned} \mu_i(f,t) \quad = \quad & \{\emptyset\} \cup \{\{\sigma\} : \ \sigma \in \Sigma_i \ \wedge \ s(m_{\Sigma_i} \circ f|_{[0,t)})\sigma \in L_i \\ & \wedge \ \sigma \in \varepsilon_i(f|_{[0,t)}, t) \setminus \eta_i(f|_{[0,t)})\ \}. \end{aligned}$$

The empty output event can always occur, therefore always $\emptyset \in \mu_i(f,t)$. The second term of expression has four subterms in conjunction. The second one ensures that only logical observations in L_i are generated. The third one is responsible for satisfaction of least delays. And the last one allows to generate only event labels enabled by means of control.

Considering the above given definitions, an EDES corresponding to a given RW-System is a TDES $\mathcal{S}_i = (\mathbb{R}_0^+, \leq, U_i, \Sigma_i, \mathcal{G}_i)$, where

$$\mathcal{G}_i = \{f \in \mathcal{M}_i : \ \forall t \in \mathbb{R}_0^+, \ m_{\Sigma_i} \circ f(t) \in \mu_i(f,t)\ \} \quad .$$

B. Some Results

The following results which can be found elsewhere are provided here for convenience.

Theorem B.1: [8] Let $\mathcal{Z}$ be a controllable behaviour of $\mathcal{S}$. For all $f \in \text{pref}(\mathcal{Z})$ and $y \in m_\Sigma \circ \mathcal{C}_\mathcal{G}(f)$ the following condition

$$\forall u \in m_U \circ \mathcal{O}_\mathcal{Z}(f), y \in m_\Sigma \circ \text{pref}(\mathcal{O}_\mathcal{G}^U(u))$$

implies that $y \in m_\Sigma \circ \text{pref}(\mathcal{O}_\mathcal{Z}(f))$. ■

Theorem B.2: [5, 13] Let $\mathcal{S} = (T, \leq, U, \Sigma, \mathcal{G})$ be an ordinary TDES constructed using an output law μ_o. Let μ_c be a strong control law of $\mathcal{S}$. Let $\mathcal{Z} = \{f \in \mathcal{G} : \forall t \in T, m_U \circ f(t) \in \mu_c(f,t)\}$. The following statement

$$\forall f \in \text{pref}(\mathcal{G}), \ (\ \forall t \in \text{Dom}(f), m_U \circ f(t) \in \mu_c(f,t)\ ) \ \Rightarrow \ f \in \text{pref}(\mathcal{Z})$$

implies that $\mathcal{Z}$ is a strongly controllable behaviour of $\mathcal{S}$. ■

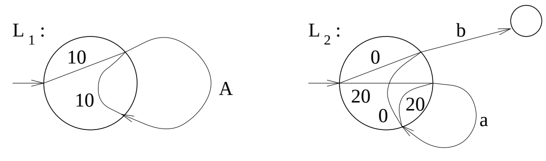

Figure 1:

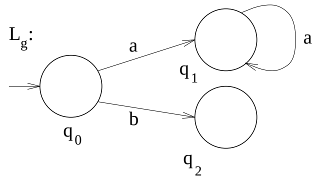

Figure 2:

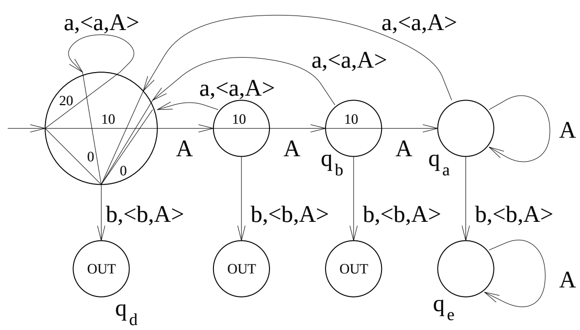

Figure 3: