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RESEARCH REPORT

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Experiments with LQ adaptive controller in the heat exchanger station at STU Bratislava

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1 Introduction

While the theory of LQ controllers represents the basic issue of a modern control theory, the practical applications are much more rare. The same is true for the adaptive version of LQ controller. Within the Copernicus project CP-1174 it was planned to test and analyze the behaviour of modern predictive model based controllers in the real plant. To accomplish this goal colleagues from the Mechanical Faculty of the Technical University Bratislava prepare the experimentation on the heat exchanging station of the university. A PC computer was connected to the existing control system. Automatic switching mechanism was build up to switch on the standard controller in case the experiment leads to the undesirable behaviour. In this way the problems with the technology as well as operators of the plant were safely prevented. The Matlab -Simuling environment was used for the communication with the technology. Various predictive controllers were tested in the plant. This report describes the experiments with LQ adaptive controller.[1]

1.1 The technology

Trials were carried out on a standard industrial heat exchange station at the university campus. The station supplies heat and hot water to all university buildings and facilities. It is equipped with a two-stage tubular concentric counterflow exchanger having 20 m^2 of the heat-exchanging surface. The temperature of the outlet is controlled by varying the flow of the primary hot water by means of a servo valve. The dynamics of the exchanger can be roughly approximated by a first-order system with the time delay about 30 s and with the rise time about 120 s. [2]

In the existing setup, the plant is controlled by a PID controller running with the sampling rate of .5 s. and with the set point for the outlet temperature of 55^oC. The PID constants are as follow:
P= 2, I=.003, D=0.

1.2 LQ controller

The LQ controllers belong to classical modern control theory. Their use in practice is often considered as difficult due to the limited possibility to cope with restrictions which practice imposes on the control loop behaviour.

[1]This research is also partially supported by the project GA ČR 102/97/0118.

[2]This text was used with the permition of authors from the paper: Experimental verification of stabilizing predictive control, by P. Zelinka, B. Rohál-Ilkiv and A.G.Kuznetsov, submitted for IJP

This drawback can be, however, circumvented by various simple trick that makes the controller more flexible but retains all advantages of the LQ approach, namely, its stability guarantees, optimality behaviour and known robustness properties.

Let us briefly remind basic features of the LQ approach used here with some suggested tricks [1].

1. Model of the process is supposed in the form of a regression model

$$y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{i=0}^{n} b_i u(k-i) + e_k + \sum_{i=1}^{n} d_i v(k-i) + K \qquad (1)$$

This model describes the dependence of output on three signals, namely, the system input $u(t)$ disturbance $e(t)$ and the measurable disturbance $v(t)$. The equation (1) defines three transfer functions

$$\frac{y}{u} = \frac{B}{A}; \ \frac{y}{e} = \frac{1}{A}; \ \frac{y}{v} = \frac{D}{A}$$

2. The control quality is evaluated by the criterion

$$\Psi = \sum_{i=k+1}^{k+T} [f_y(w(i) - y(i))^2 + f_u(u(i) - u0(i))^2]. \qquad (2)$$

It penalizes a function of the control error between the actual and desired output and a function of the difference between the actual input and some reference $u0$. The introduction of this reference is a novelty and will be discuss later.

This form of the criterion widens substantially the possibilities of controller tuning and reflects the practical situation when quite often a function of input or output is to fulfill given specifications [2],[3].

3. The minimization of the criterion (2) based on the model (1) is based on dynamic programming in a state space. The nonminimum dimension state space is used here, using delayed measured data as elements of the state space. This choice increases only moderately the computational burden of the square root minimization algorithm simplifying to a great deal the interpretation of results.

4. A reliable square root algorithm for the minimization of the criterion (2) allows easily implement various types of functions f_y, f_u considered in the criterion.

5. The IST (iteration spread in time) strategy makes the adaptive version of the approach extremely fast.

6. To be able to cope with typical signal constraints the controller algorithm enables an automatic penalization adjustment based on input signal and its moves constraints [4].

7. The use of feed forward signal from measurable disturbance is known to be very efficient way of compensation.

8. The introduction of a input reference signal has a multi-purpose role.
a. That signal is necessary in cases we did not use the incremental model. The liner model used is only an approximation around the plant working point. While it is standard to compare $y(t)$ with $w(t)$ nothing similar is done with the input variable. The introduction of $u0$ is removing this drawback
b. The signal $u0$ can be considered as another free variable to be used for the minimization of the criterion (2). [5]. Typically it can be used to make control error zero in specific future time.
c. As it can be seen from the form of the criterion, its symmetry now suggest that the penalization ratio Qu/Qy is now a knob tuning the controller behaviour between the set point reference following and input reference following. This form is extremely useful in introducing another controller (let us say reference controller) to the loop.

Standard on-line LS algorithm with exponential forgetting was used for system parameter identification.

1.3 Programming aspects.

All the features listed above has been implemented into the Matlab and Simulink functions. Basic Simulink schemes used for the direct control of the plant are shown in Figure.1 and Figure.2. The first one realizes the positional control, the second one adds an integrator to the loop. The schemes representing a "cheap" controller where the penalization of the output depends on the predicted control error is on Figure.3. The output of the block is system input $u(t)$, the input points `in_1` to `in_6` are places where the following signals enter
`in_1` – system input $u(t)$,
`in_2` – system output $y(t)$,
`in_3` – setpoint $y0(t)$,
`in_4` – output level where the output starts to be penalized,
`in_5` – minimum penalization (when no additional penalization is used)
`in_6` – the pair of $[u(t), y(t)]$ for parameter estimation

Figure.4 shows the block diagram of the arrangement for data dependent input penalization adjustment. Here the inputs points of the block serve for:
`in_1` – system input $u(t)$,
`in_2` – system output $y(t)$,

`in_3` – setpoint $y0(t)$,
`in_4` – reference input $u0(t)$,
`in_5` – minimum penalization (when no additional penalization is used)
`in_6` – the pair of $[u(t), y(t)]$ for parameter estimation

As the setting of penalizations are in both cases totally independent it can be simply implemented together.

Figure.5 shows the way of using the reference input signal. The signal can be either set to some value, or it can be hanged to the output of other controller.

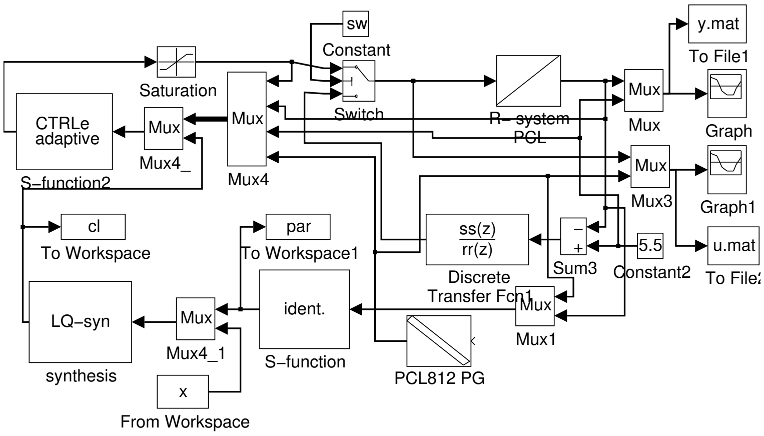


Figure 1: Basic control scheme

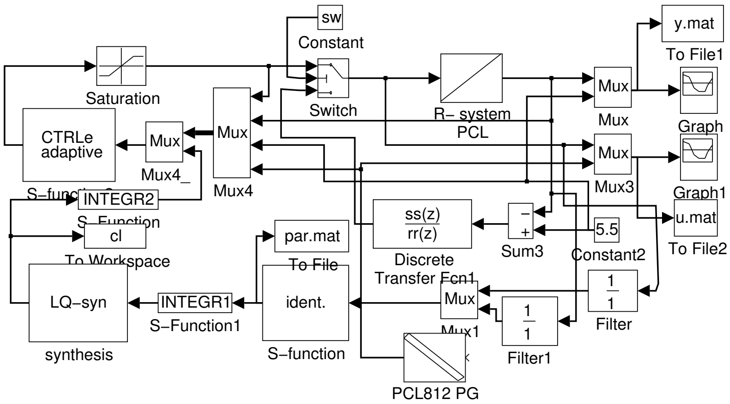


Figure 2: Basic control scheme with incremental controller

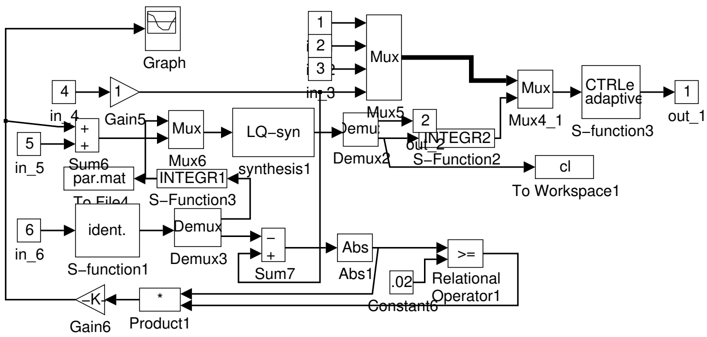


Figure 3: Adaptive LQ controller with output penalization depending on the output prediction.

2 Experimental results

Experiments were caried out in three periods of the year representing different operating conditions of the technology.

The aim was to test

- behaviour of the LQ controller, i.e.
  - how the input penalization influences the behaviour
  - what influence has the structure of the model(controller)
  - verify the possibilities of more sophisticated penalizations
- adaptation process
- startup of the adaptation
- ways to ensure the smoothness of the transition from the standard control to the LQ and back

The first experiments were carried out in the summer period. The primary water temperature was low (70^oC) and hot water consumption was rare. To see the controller actions an artificial disturbances were created by opening the hot water tap in nearby lavatory for approximately a minute. Several experiments run on the heat exchanger. Different models , sampling and penalization was tested in these cases. Using the prior information about the plant the 2nd order regression model with 2 steps of time delay for the sampling period $T_0 = 10s$

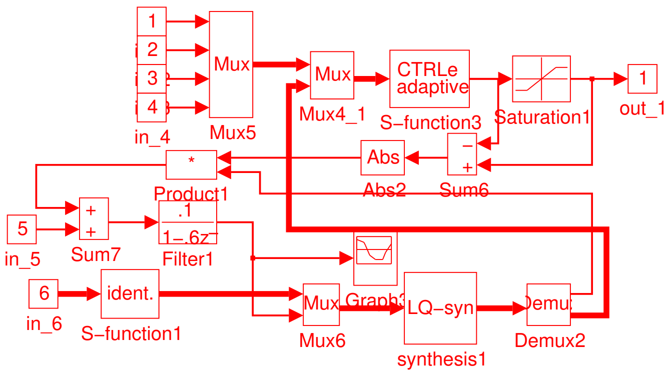


Figure 4: Adaptive LQ controller with input penalization depending on constraints

was used as the initial one. By a few experiments it was shown that this model is quite satisfactory and a reasonable penalization was from the interval $Qu = (.1 - 1)$

Figure.6 shows the situation when the adaptive controller start in a smooth way from the standard PID controller of the plant. The dropouts in the input variable were caused by stopping the simulation to change penalization Qu. Then the simulation started with zero initial conditions. The output variations were caused by a disturbance. This can be deduced from the fact that both the PID and adaptive controllers tend to change input value at the same direction. When the dropouts occurred PID controller moved in the opposite direction.

Model: 2nd order model without delay. — Final estimation of parameters:
B: 0.0128 -0.0155 0.0083 A:1 -1.3116 0.3148
roots(A) 0.9953 0.3163; roots(B): 0.6055 + 0.5309i, 0.6055 - 0.5309i
Final controller: R:1.0000 -0.1004 0.1047 ; S: 12.9625 -3.9731
Qu: 100 $\rightarrow$ .05

Figure.7 shows rather long run with a first order model with three steps of delay, by the mistake the parameter a1 was fixed to zero. This model is not fully correct. It estimates the constant term in the regression model but did not used it for control. This could be a substantial reason for the steady state error.
Last estimates:
Model: B: -0.0270 0.0209 -0.0003 0.0043 0.0131 A: 1 0 - 1.1004 ;
roots of B: 1.1771; 0.1203 + 0.7911i; 0.1203 - 0.7911i; -0.6437
roots of A: 1.1004

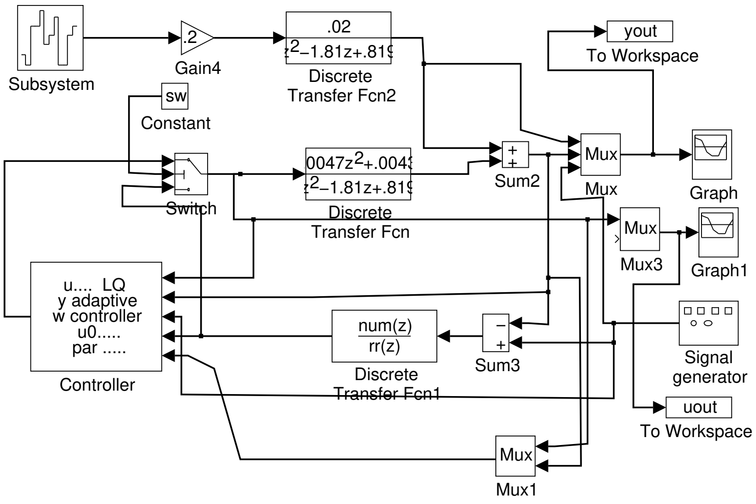


Figure 5: Adaptive LQ controller with PID

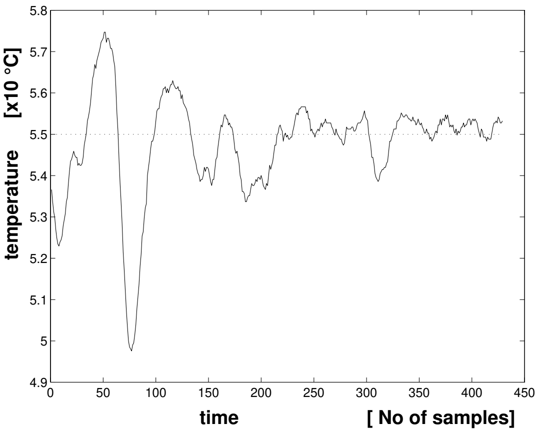
Controller: R: 1.0000 0.4321 0.2713 0.2941 0.1098 S: 20.2989 9.9562

Figure.8 shows again nice startup caused by smooth tuning from the standard PID controller and full LQ controller.

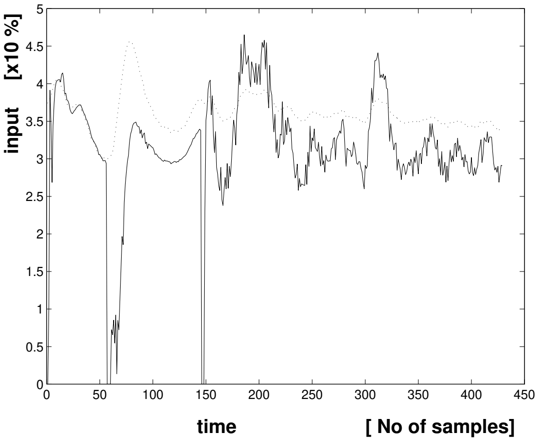
Figure.9 shows long run over 1000 samples. The input penalization varied from a large value Qu= 1 down to Qu= .001 and later up to Qu=.1. The graph shows different behaviour throughout the experiment. The model now was full second order with two steps of time delay. It is interesting to see the estimation parameters change (Figure.14b). The A parameters went slowly down but the disturbance changed them abruptly. Sampling period 10s. Last estimates:
Model: B: -0.0003 -0.0018 0.0005 0.0008 0.0041 A: 1 - 1.4769 0.4790
roots of B: -6.2542; 1.4092; -0.5775 + 1.1032i; -0.5775 - 1.1032i
roots of A: 0.9960 ; 0.4809
Controller: R:1.0000 0.0668 0.0975 0.0868 0.0718 S: 17.1811 -8.3065

Figure.10 documents the run with a model sampled at 5s rate. The run was very stable without disturbances. Also parameters were not changing much (Figure.15a).
Final estimates:
Model: B:0.0105 0.0059 -0.0075 -0.0182 0.0221 A: 1 -0.6877 -0.3058
roots B: -1.1380 + 0.9784i; -1.1380 - 0.9784i; 0.8570 + 0.4473i; 0.8570 - 0.4473i
roots A: 0.9950 -0.3073

Figure.11 shows the control experiment using LQ controller with integral action. It is seen how it keeps the desired value in the mean, however, for given penalizaton of u(t) its input moves too much.
Model: Sampling 10s. 2nd order with 2 steps of time delay.

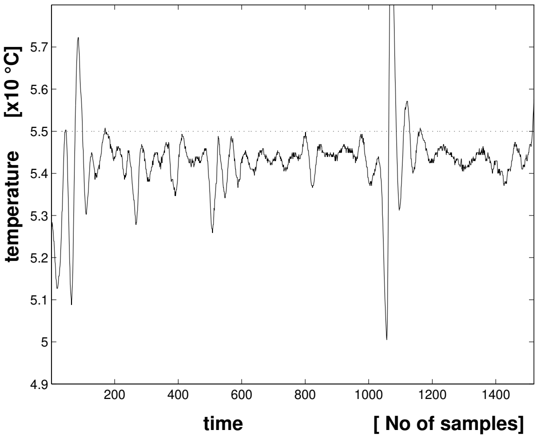


6a. output and setpoint

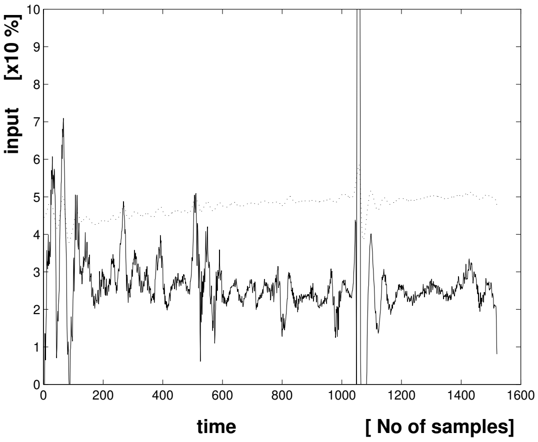


6b. input

Figure 6: Experiment No 1.

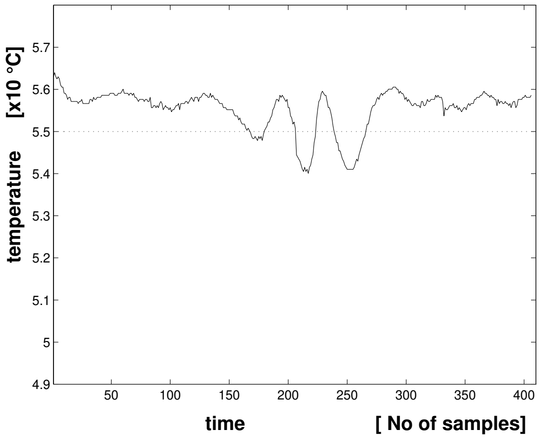


7a. output and setpoint

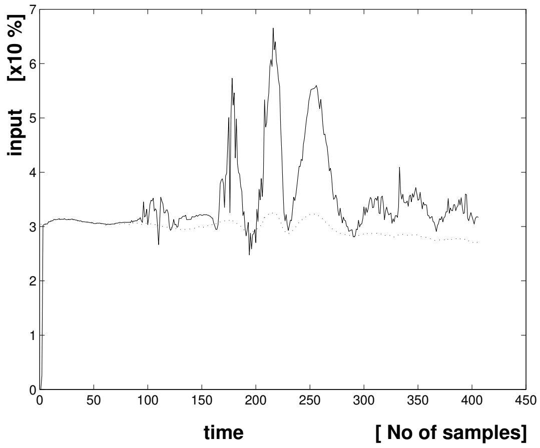


7b. input

Figure 7: Experiment No 2.

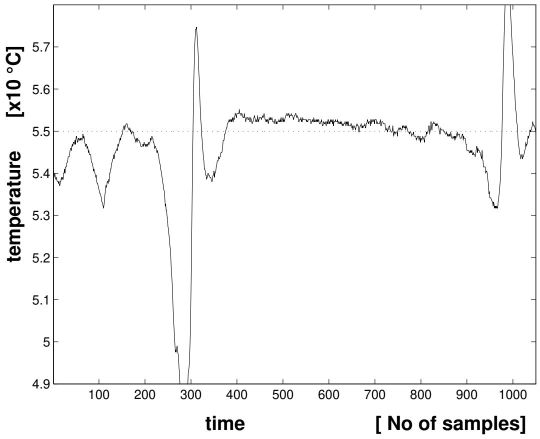


8a. output and setpoint

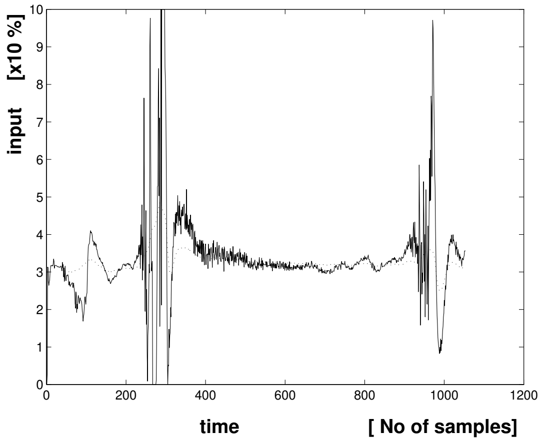


8b. input

Figure 8: Experiment No 3.

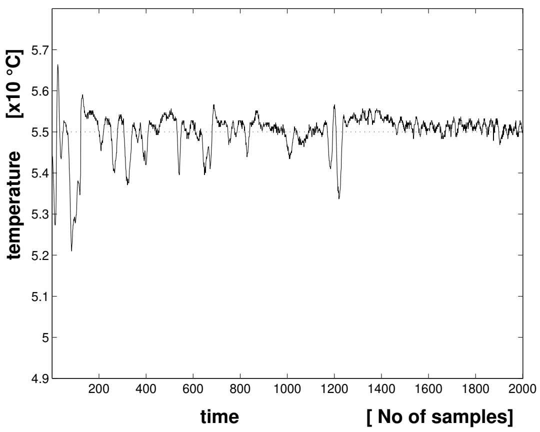


9a. output and setpoint

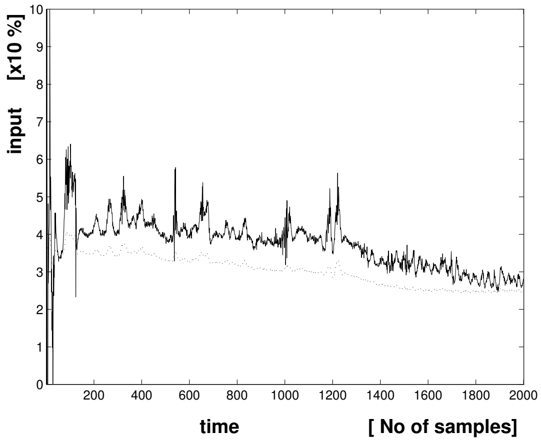


9b. input

Figure 9: Experiment No 4.

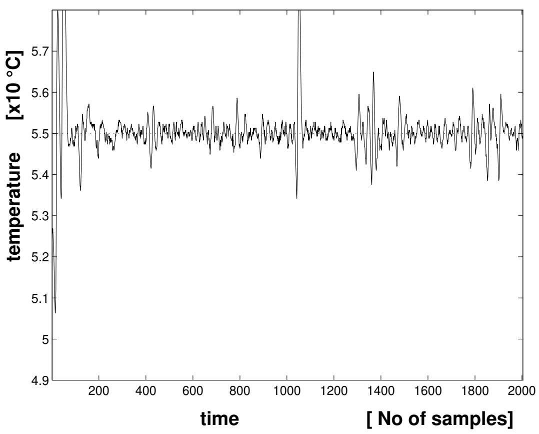


10a. output and setpoint

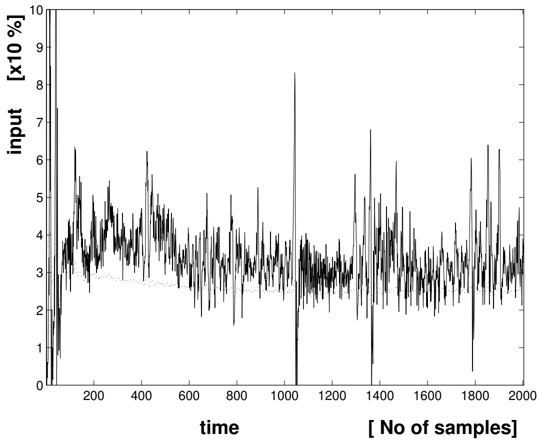


10b. input

Figure 10: Experiment No 5.

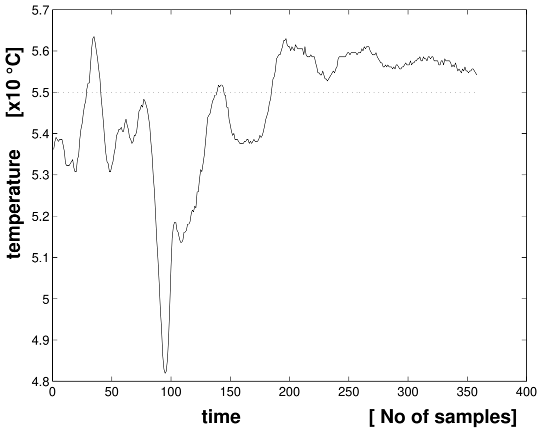


11a. output and setpoint

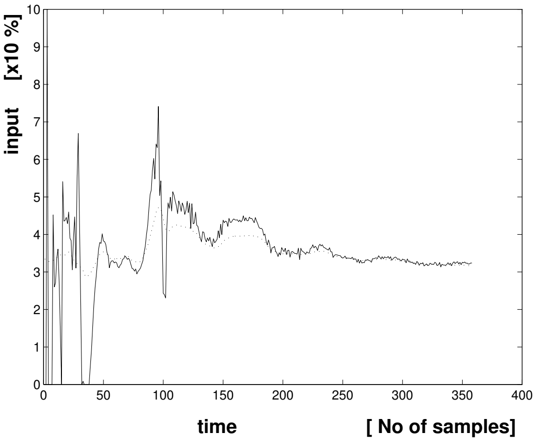


11b. input

Figure 11: Experiment No 6.



12a. output and setpoint



12b. input

Figure 12: Experiment No 7.

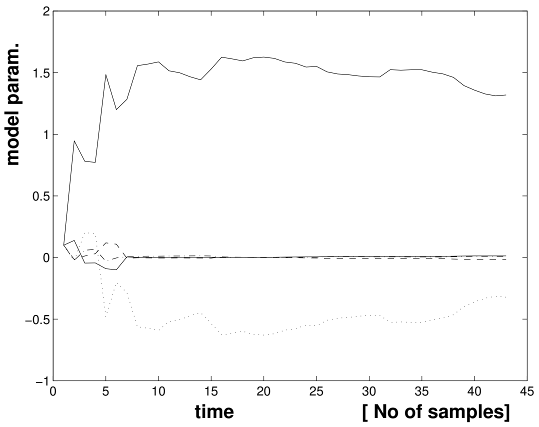
last estimates:
B: 0.0016 -0.0044 -0.0004 0.0038 0.0057 A: 1 - 1.2574 0.2612 roots of B: 2.0375 + 0.4174i; 2.0375 - 0.4174i; -0.6625 + 0.6203i; -0.6625 - 0.6203i
roots of A: 0.9948 0.2626
Control law for Qu=.1;
R:1.0 0.0449 0.4643 0.4227 0.2725
S:57.1531 -72.9818 18.4648

Figure.12 shows the experiment with LQ controller where the penalization of Qu=.1 gives the moves of the controller similar as PID . After parameters were adjusted the controller more or less copies the action of PID.
Model: Sampling 10s, second order with 2 steps of time delay.
Last estimates: B: -0.0013 -0.0112 0.0007 0.0096 0.0059 ; A: 1 -1.2691 0.2714;
roots of B: -8.5851; 1.1134; -0.5718 + 0.3844i; -0.5718 - 0.3844i
roots of A: 0.9968; 0.2723
Conrol law:
R: 1. 0.0209 0.1018 0.0877 0.0070; S: 7.3025 -3.6723 ;

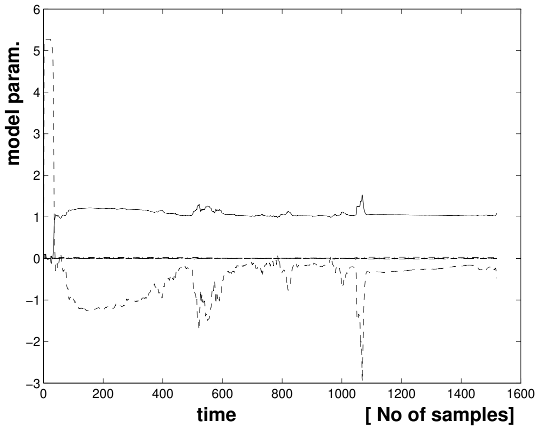
Figure.13 till Figure.16a shows the parameter estimates of the experiments. Figure.16b shows the change of Qu for experiments No 4 and 6.

It is worth noticing that in all experiments the identified parameters, whatever they were give the dominant time constant around .99. Also it is seen that in most cases the numerator of the model was nonminimum phase. Quite often with several nonminimum phase roots.

The following three experiments were performed later when the station was in full load. All models second order with time delay and use sampling period 10s.

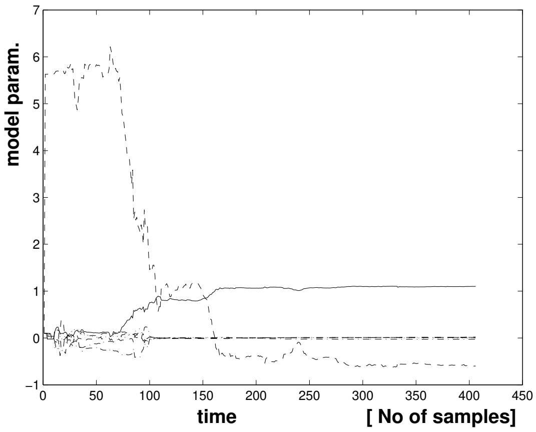


13a. 1. exp.

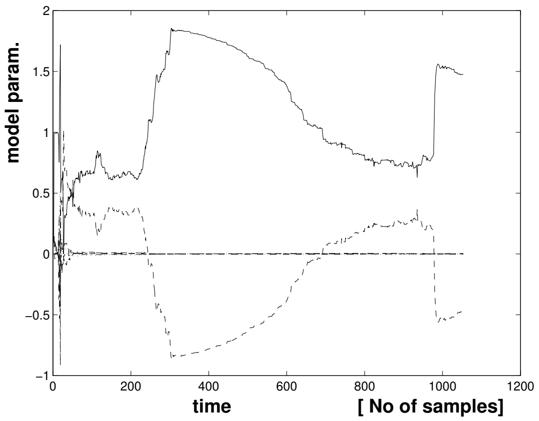


13b. 2. exp

Figure 13: Parameters .

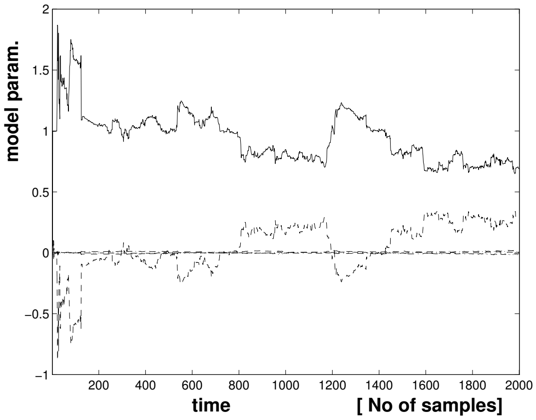


14a. 3. epx

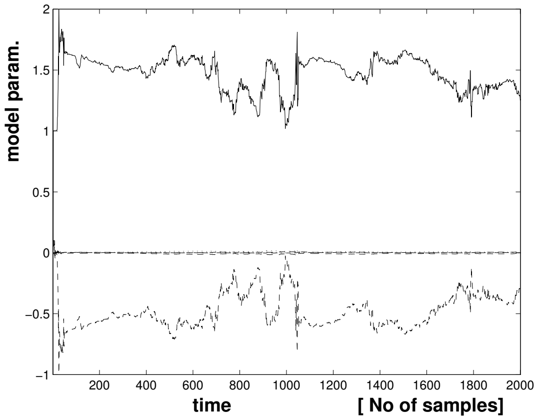


14b. 4.exp

Figure 14: Parameters.

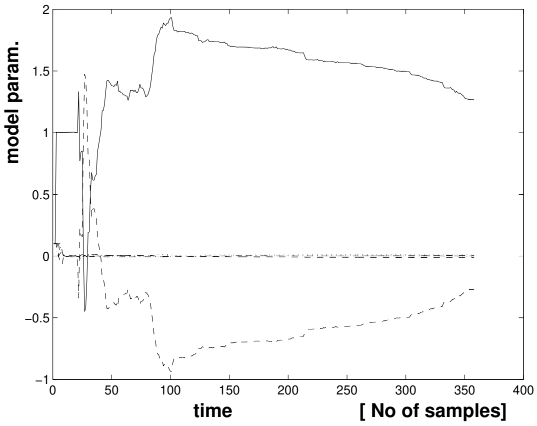


15a. 5.exp

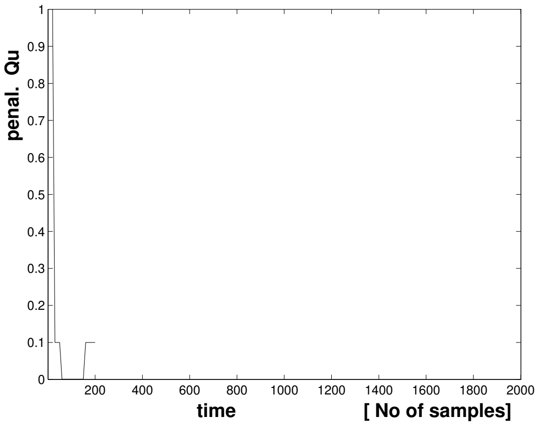


12b. 6.exp

Figure 15: Parameters

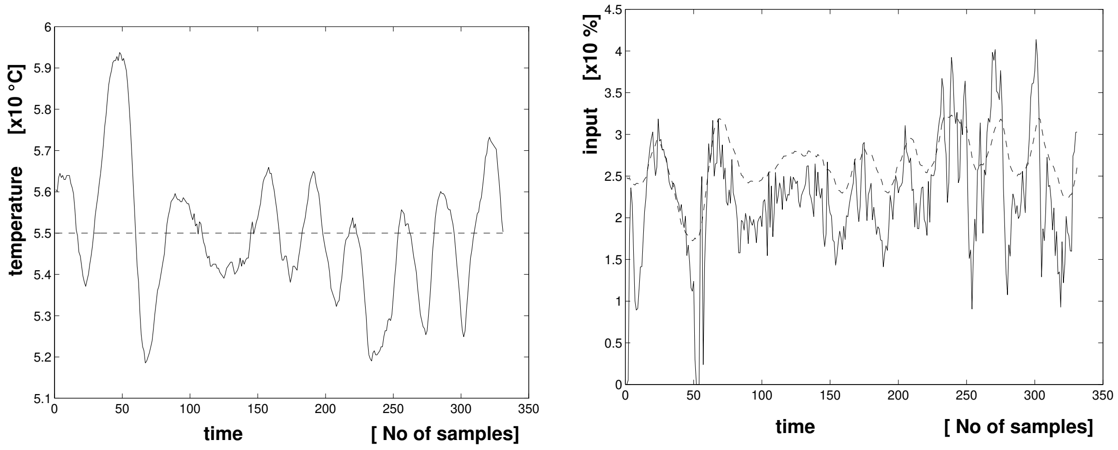


16a. Parameters 7.exp



16b. input penalization for 4.and 6. exp

Figure 16: Mixed results



17a. output and setpoint

17b. input

Figure 17: Experiment No 8.

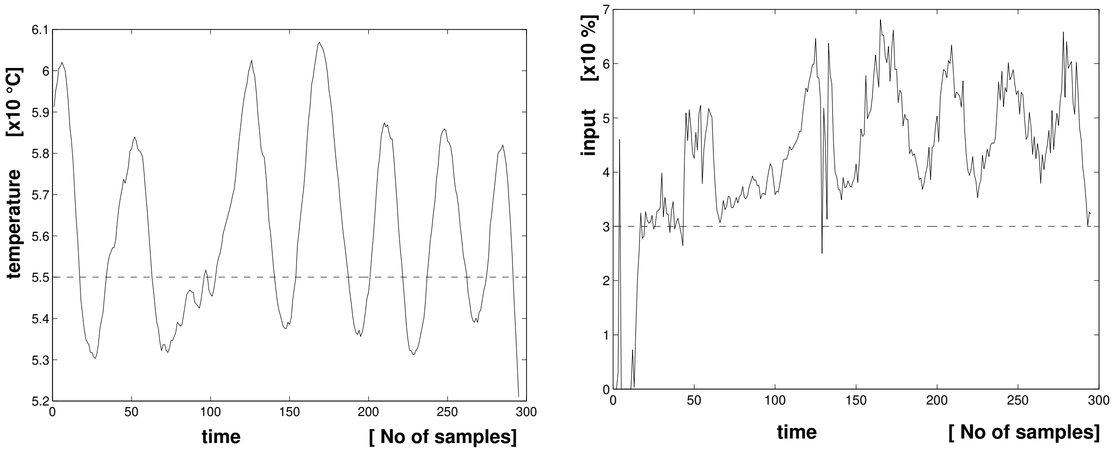
Figure.17 shows the smooth transition between PID behaviour and LQ setup. Again there is an obvious that the output changes are due to the disturbance as the actions of both controllers are in the same direction.
Model: 5 steps of time delay. Last parameter estimates
B:0.0061 -0.0096 0.0044 -0.0017 0.0063 -0.0042 0.0034 0.0034
A: 1 -1.7461 0.7496
roots of A: 0.9854 0.7607
roots of B: 1.1759 + 0.5230i; 1.1759 - 0.5230i; -0.5741 + 0.7503i; -0.5741 - 0.7503i; 0.4167 + 0.8000i; 0.4167 - 0.8000i; -0.4634

The evolution of parameters is documented in Figure.20a.

Figure.18 shows the control experiment with fixed input reference. The model suppose time delay=5. The estimates of the model parameters at the end of experiment are
B: -0.0029 0.0074 -0.0050 0.0038 -0.0015 0.0003 -0.0045 -0.0032
A: 1 -1.8150 0.8103
roots B: 1.8106; 1.4090; 0.3737 + 1.0478i; 0.3737 - 1.0478i; -0.4509 + 0.6908i; -0.4509 - 0.6908i; -0.5136;
roots A: 1.0226; 0.7924;
control law :
R: 1.0000 0.0694 0.1571 0.0069 0.1456 0.0672 0.0785 -0.0138
S: -23.5521 18.9831

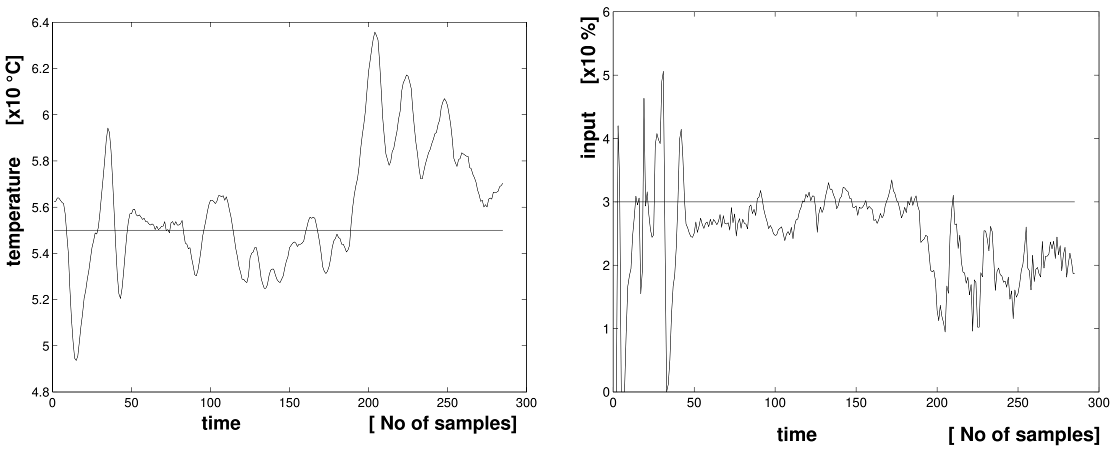
The evolution of parameter estimates is in Figure.20b.

Figure.19 documents another experiment with LQ adaptive control with fixed $u0$. Model now assumes time delay k=2. Model parameter estimates at the end are:



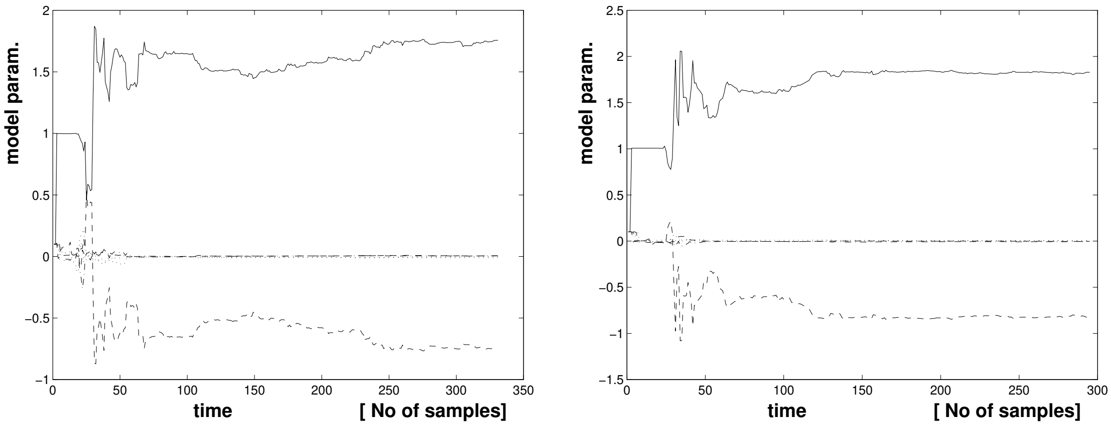
18a. output and setpoint
18b. input

Figure 18: Experiment No 9.



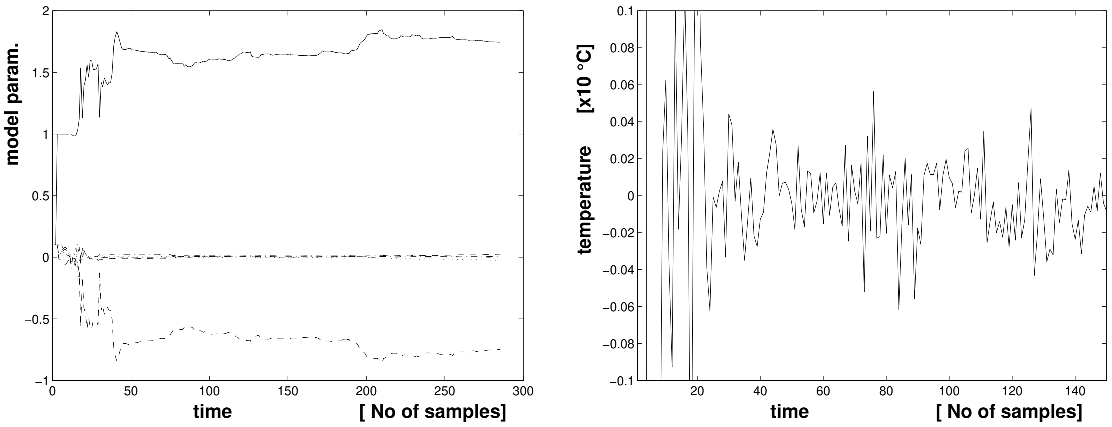
19a. output and setpoint
19b. input

Figure 19: Experiment No 10.



20a.experiment No.8 20b. experiment No.9

Figure 20: Model parameters.

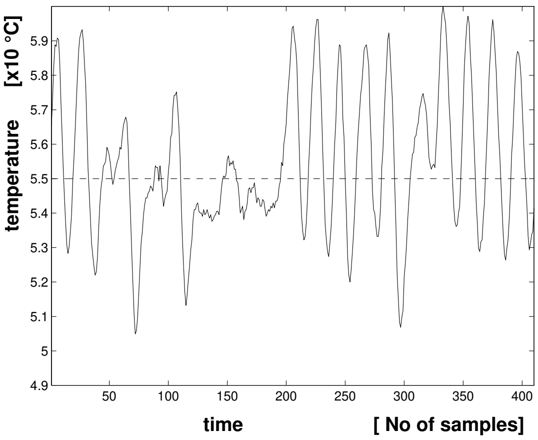


21a. Model parameters 21b. prediction error

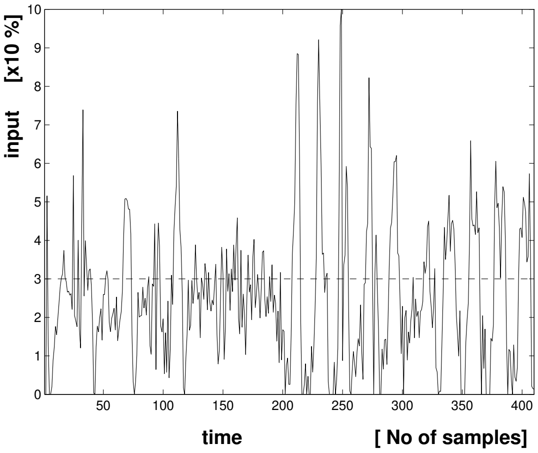
Figure 21: Experiment No 10.

B: 0.0064 -0.0235 0.0212 0.0022 0.0038
A: 1 -1.7472 0.7514
roots of B: 1.9490 + 0.4988i; 1.9490 - 0.4988i; -0.1131 + 0.3659i; -0.1131 - 0.3659i;
roots of A: 0.9821; 0.7651;
control law :
R:1.0000 0.1217 0.1670 0.1466 -0.0403
S: 10.9338 -8.6983

The last experiments were carried out in January 1998. The first example (Experiment No. 11) starts with the structure: 2nd order, 2 steps of time delay, 10s sampling period which seems to be the most appropriate structure



22a. output and setpoint



22b. input

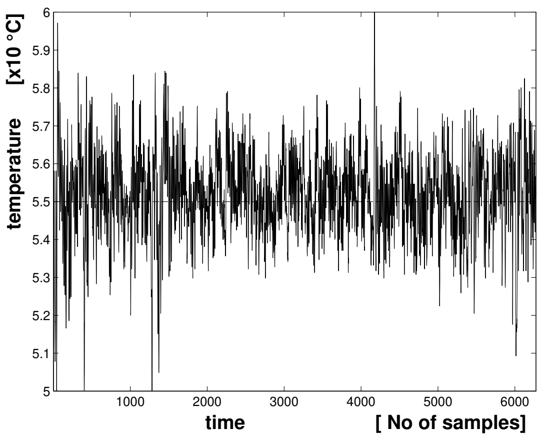
Figure 22: Experiment No 11.

from previous experiments. The results are seen in Figure.22

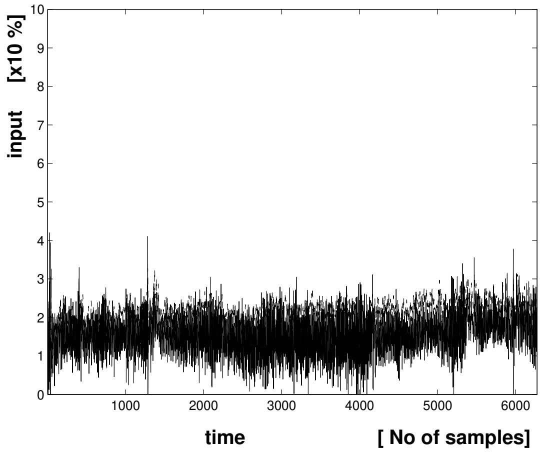
The experiment can be considered successful. The input reference here was constant and time varying penalization of the input (from Qu=100 to Qu=.2 during the first 100 steps) made the startup quite satisfactory.

As the identified model parameters gave unstable plant root, The structure: 3nd order, 1 steps of time delay, 10s sampling period was used in the next experiment (Experiment No 12), instead. This experiment starts at 16h. p.m. and lasts about 9h a.m. next day, including the evening and morning period of large disturbances. The run is quite good. The hot water consumption can be guess from the Figure.23b where it is seen how the mean value of the input variable is decreasing at the beginning and increasing again at the y7mend. An interesting form has the plot of roots of the model. During the time of relative quiet, an unstable root appears. The fast change of this root at the time after 4000 sample is accompanied by a sharp overshoot in the output. At the startup the adaptive controller followed the $u0$ generated by standard PID but as the initial penalization was not high enough (it changed from 10 to .1 in 100 steps) $u0$ was not followed strongly in this case.

Experiment No 13 started at about 9am. The aim was to verify the shorter sampling period. So the previous structure was used but with sampling $T_s = 5s$ The input and output are seen in Figure(y3,u3). The identified roots of the numerator and denominator are in Figure(citkor3,jmekor3). It can be see that

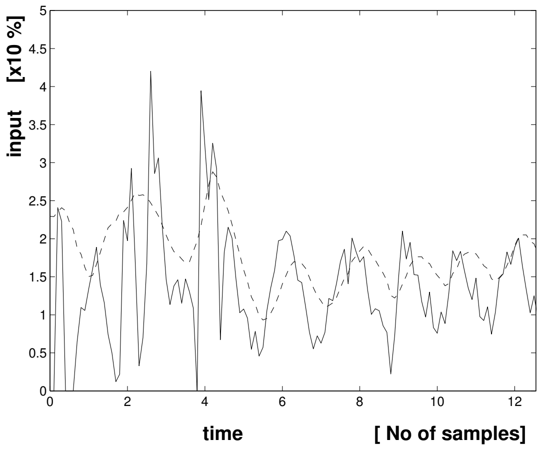


23a. output and setpoint

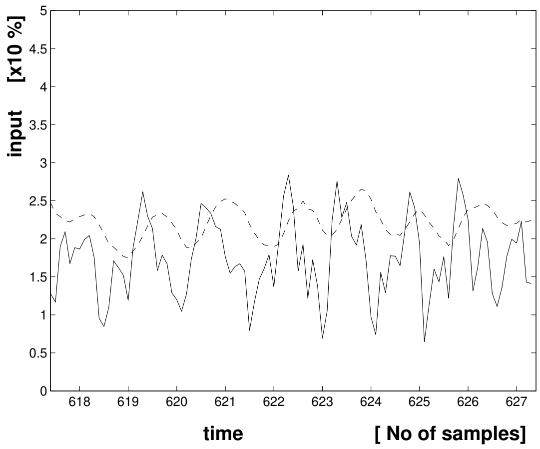


23b. input

Figure 23: Experiment No 12.

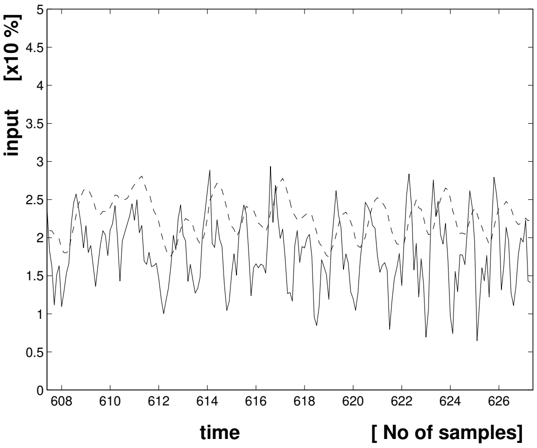


24a. input at the start

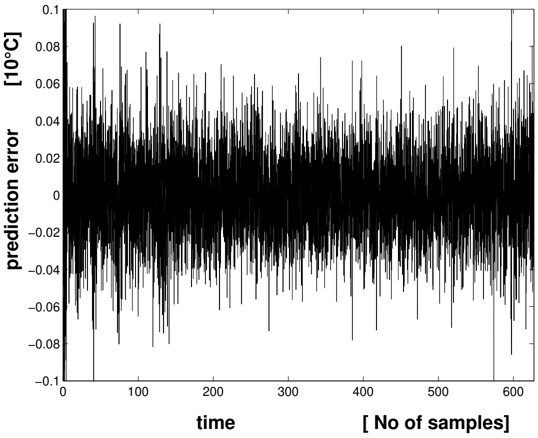


24b. input in the middle

Figure 24: Experiment No 12.

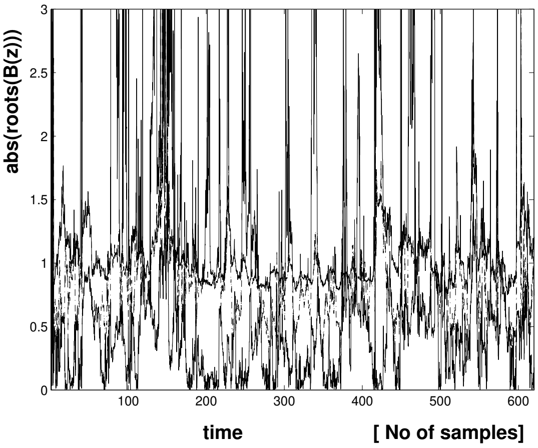


25a. input at the end

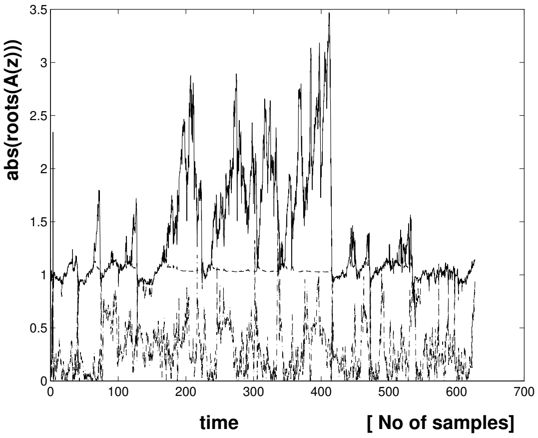


25b. prediction error

Figure 25: Experiment No 12.

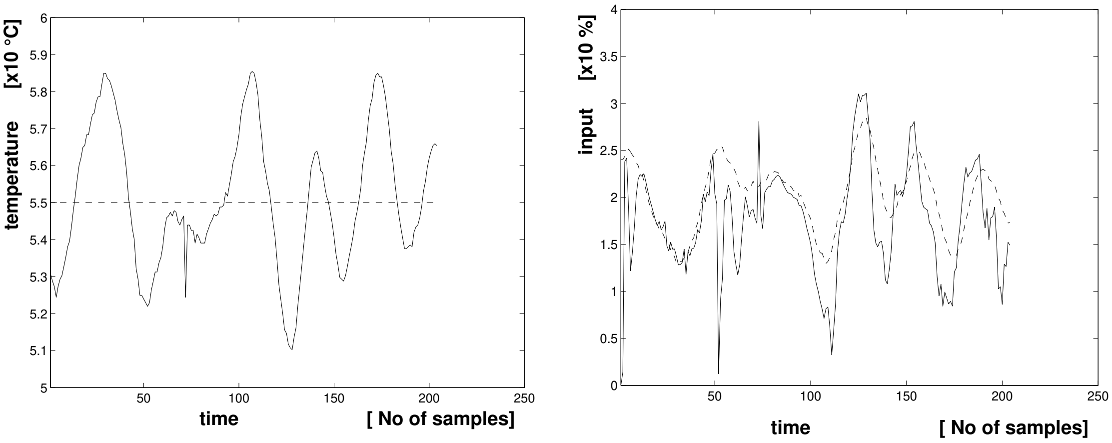


26a. roots of the numerator



26b. roots of the denominator

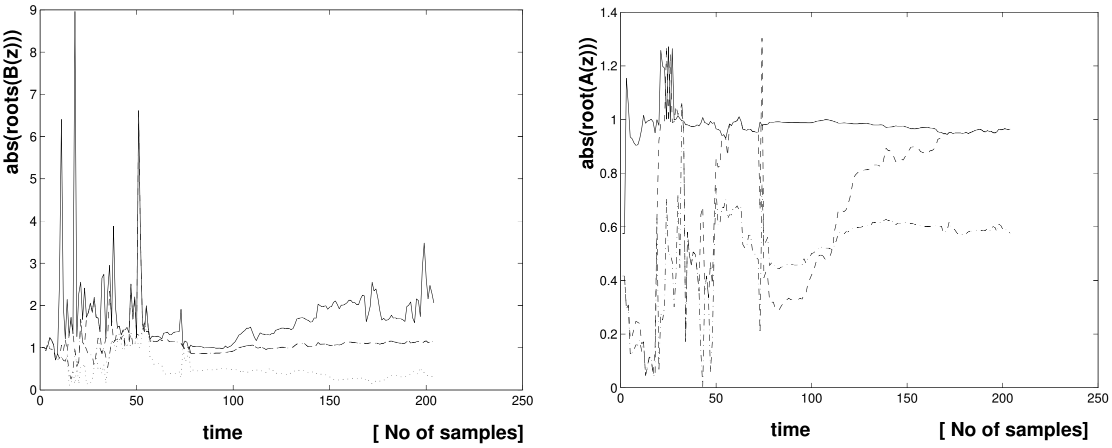
Figure 26: Experiment No 12.



27a. output and setpoint

27b. input

Figure 27: Experiment No 13.



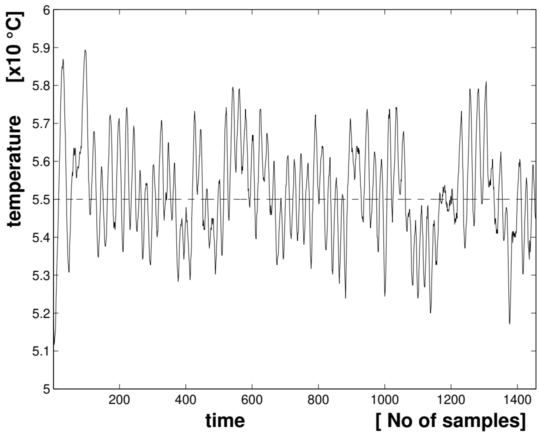
28a. roots of the numerator

28b. roots of the denominator

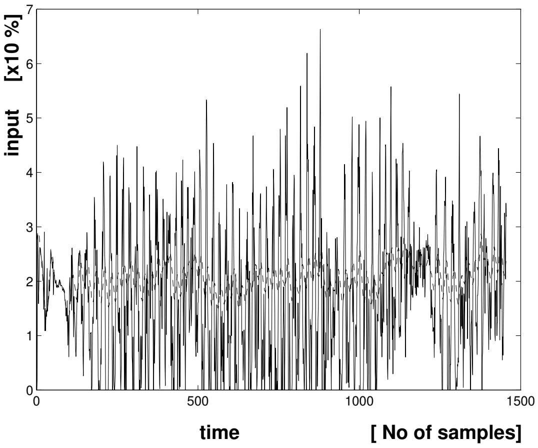
Figure 28: Experiment No 13.

after the initial movement of roots the numerator settled in three unstable roots. The startup now was done with the Qu going from 100 to .1 so after a few initial steps the adaptive controller output follow the PID signal. Mean while the estimates settle which make the startup very smooth. Unfortunately the run was cut by wrong manipulation with computer.

Experiment No 14 was the continuation of the previous one but from the usual initial conditions. It lasted more then 2 hours and contained rush hours at the lunch time. The temperature control is very good, the input is rather oscillating. In Figure.30a is documented very quiet startup from zero initial conditions. Again, as in almost all cases the model has several unstable zeros.



29a. output and setpoint

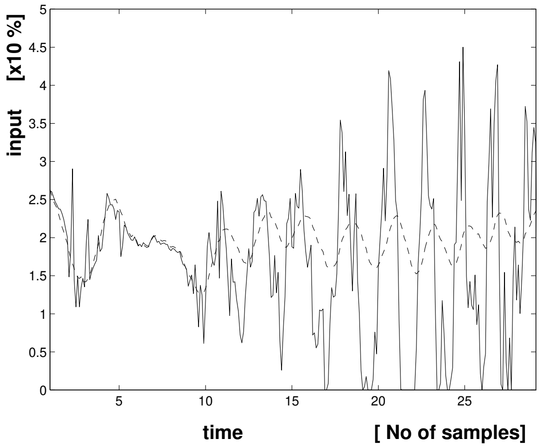


29b. input

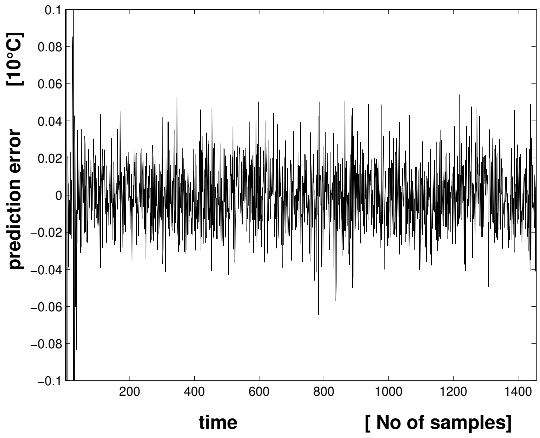
Figure 29: Experiment No 14.

Experiment No. 15 demonstrate the attempt to use the the adaptive controller with integrator and with data dependent penalization Qy so that the penalization increases in case of large control errors and is set to (almost) zero in the range of output that can be considered as satisfactory. The experiment was unsuccessful. In a short time after the start the input grew too much, the temperature exceeded 65^oC and the automatic safety device switch over the adaptive controller to the PID one. Figure.32 show the initial part, where the switching take place. Figure.33 shows the input and output of the plant with the standard PID controller.

When the other experiments with this scheme failed, the scheme was modified to a case when the controller did not contained the integrator part. While with the standard quadratic criterion this type of control leads to inputs having the form of isolated impulses which is usually undesirable in practice, the case when $u - u0$ is penalized leads to the situation that in cases when the output is within the admissible range the input follows the standard controller output but when the output exceeds given level, due to the change of penalization LQ controller will start to be active and could more intensively compensate the disturbance. To verify these features the scheme in the Figure.34 was used in the experiment No. 16. The model is 3rd order with 1 time delay, sampled at 5s. The outputs without penalizations lie in the interval [5.3 5.7] of the

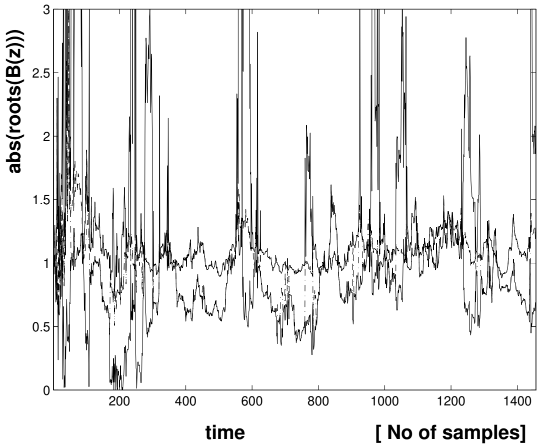


30a. input at the startup

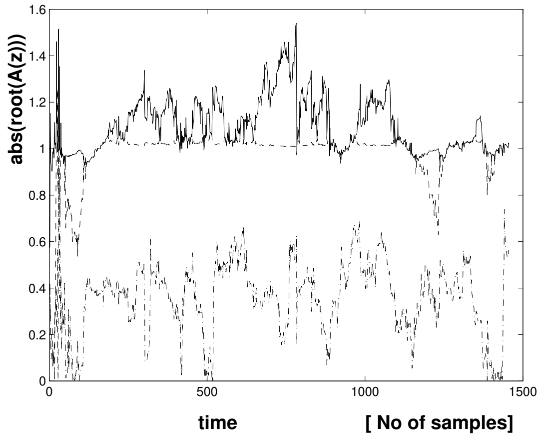


30b. prediction error

Figure 30: Experiment No 14.

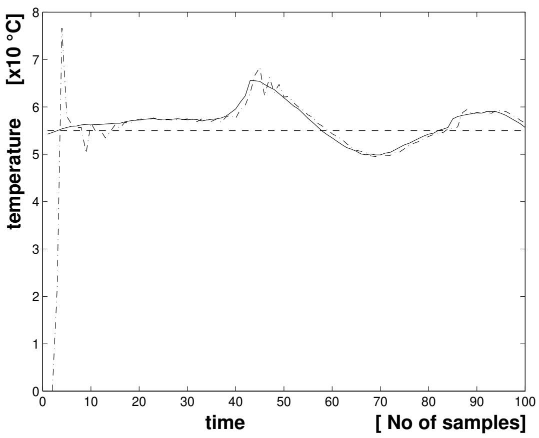


31a. roots of the numerator

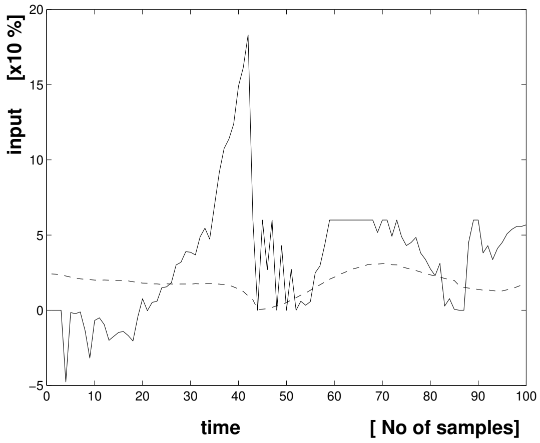


31b. roots of the denominator

Figure 31: Experiment No 14.

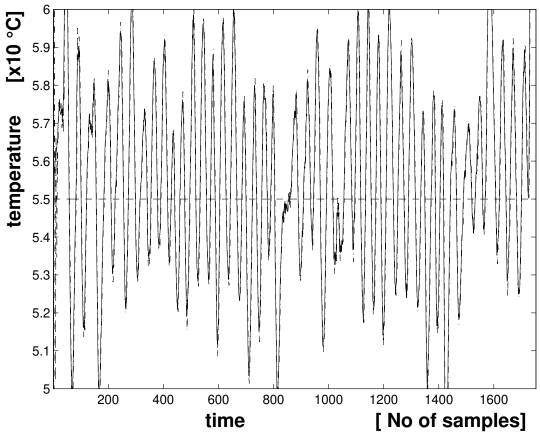


32a. output and setpoint at the beginning

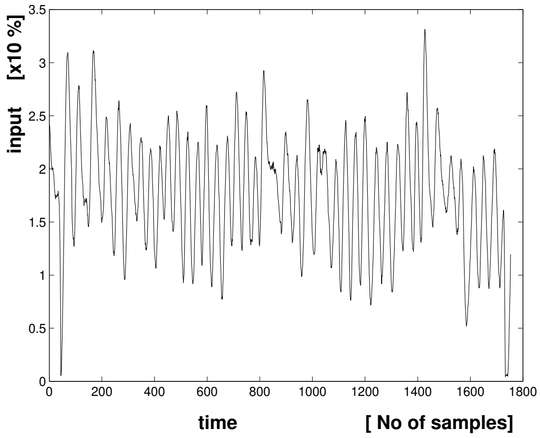


32b. input at the beginning

Figure 32: Experiment No 15.



33a. output and setpoint with PID



33b. input from PID

Figure 33: Experiment No 15.

normalized variable ([53 57]^oC). Qu=.2.

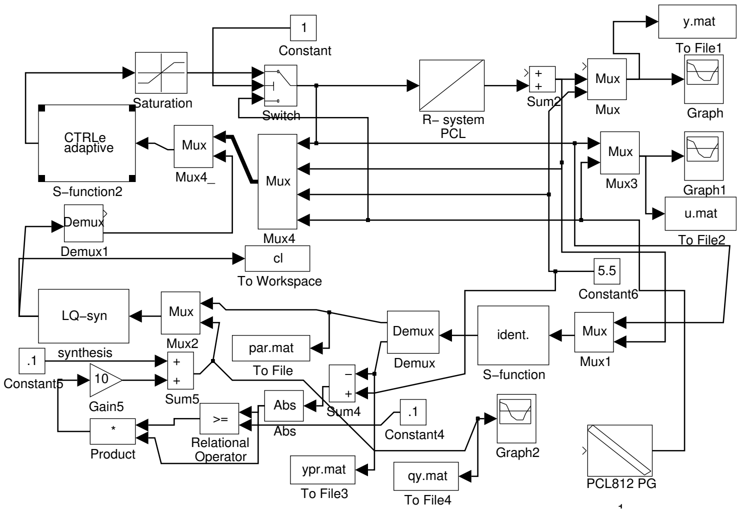


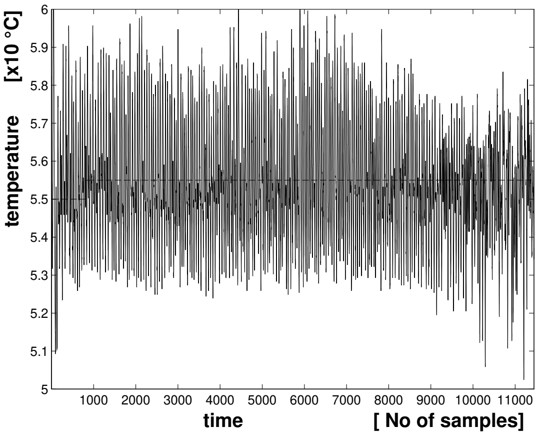
Figure 34: control scheme with variable Qy

The system input and output in a long(16h.) overnight run are in Figure.35 . The details of the input from the beginning and final part are shown in Figure.36. Time varying Qy is plotted in Figure.37 with the prediction error. The details of Qy and corresponding input in the middle of the experiment are in Figure38.

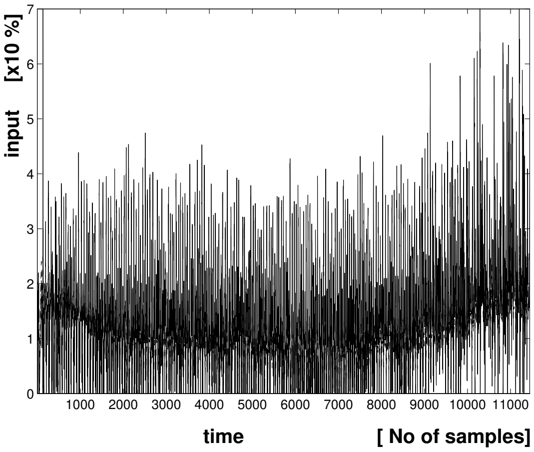
The algorithm worked properly in various conditions of the day regime. It is evident that the permanent oscillations were caused by the PID controller for these operating conditions.

Experiment No 17 is a long (5h) trial with standard build in PID controller to have a reference of usual behaviour. It spans the period from 9a.m. to 2p.m. i.e from a rather quiet time to a busy period. Figure.39 shows the output and input. The model was identified (but not used). The numerator and denominator roots show that even in these conditions the identified model was good enough to be used if necessary (see Figure.40). The prediction error is in Figure.41. It is apparent that the input changes are smaller that in some LQ adaptive controller cases, but the output quality is slightly worse.

Experiment No 18 was carried out again with a scheme in Figure.34 but

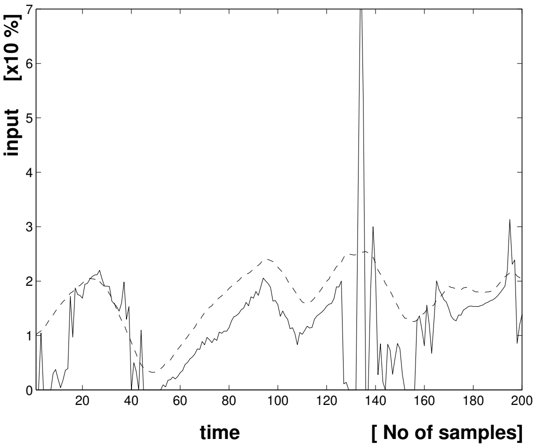


35a. output and setpoint

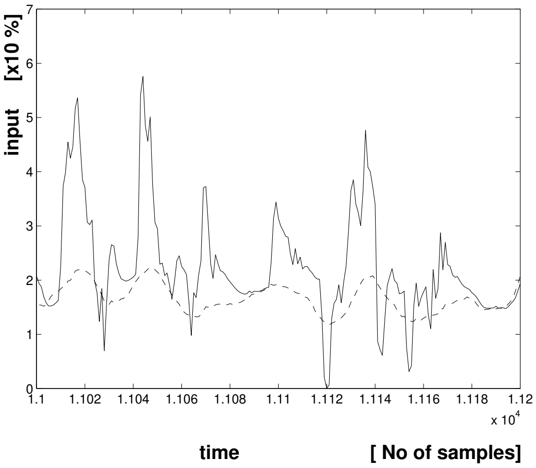


35b. input

Figure 35: Experiment No 16.

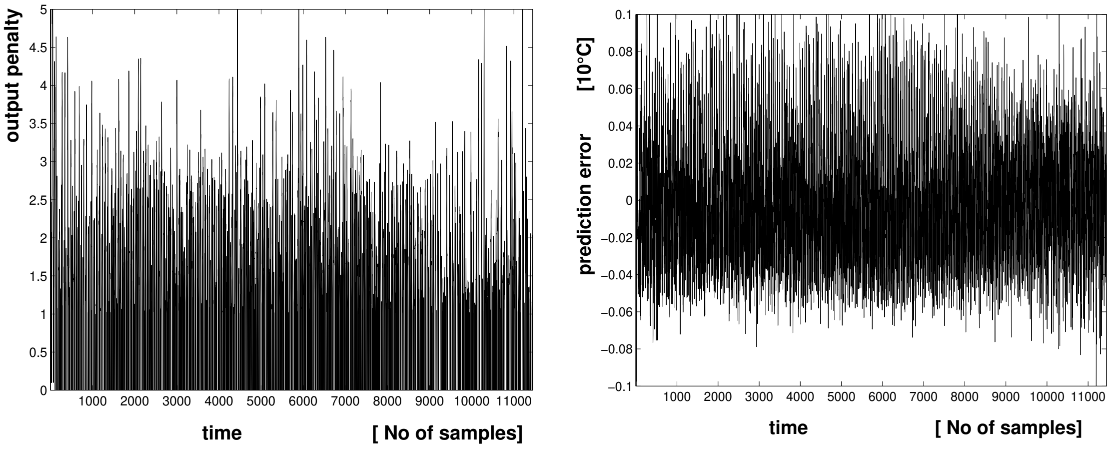


36a. input at the startup



36b. input at the end

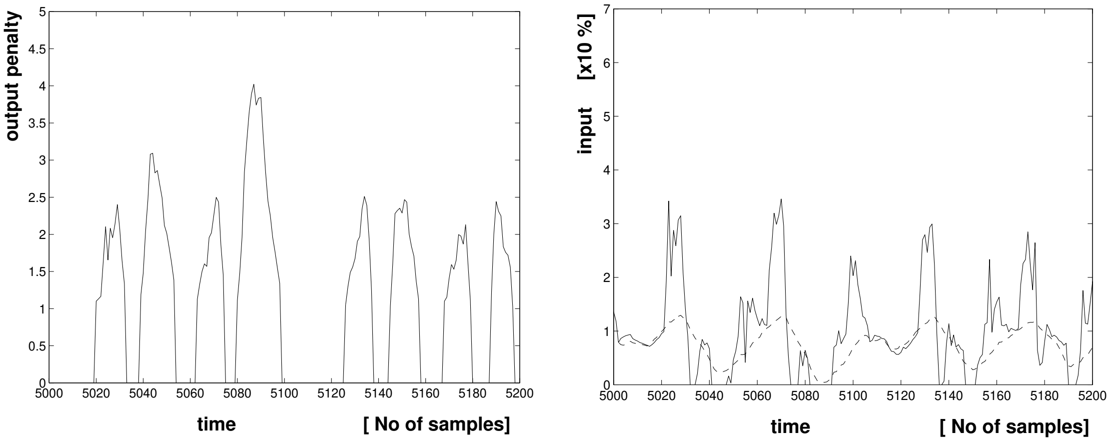
Figure 36: Experiment No 16.



37a. penalization of the output Qy

37b. prediction error

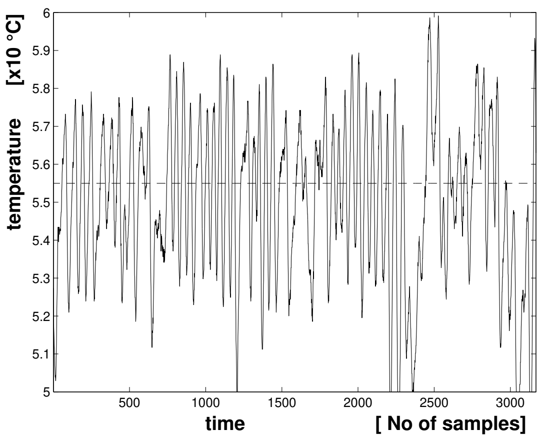
Figure 37: Experiment No 16.



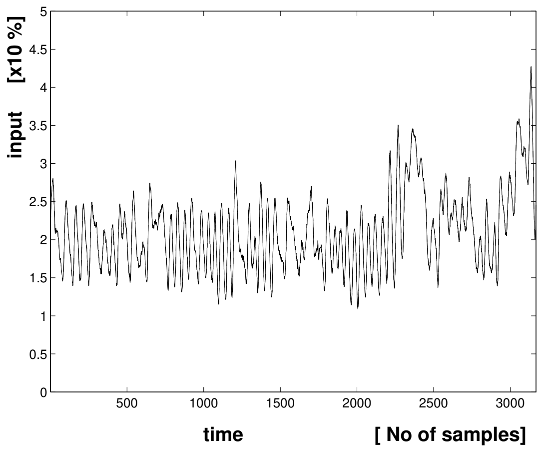
38a. details of the penalization Qy

38b. details of the input

Figure 38: Experiment No 16.

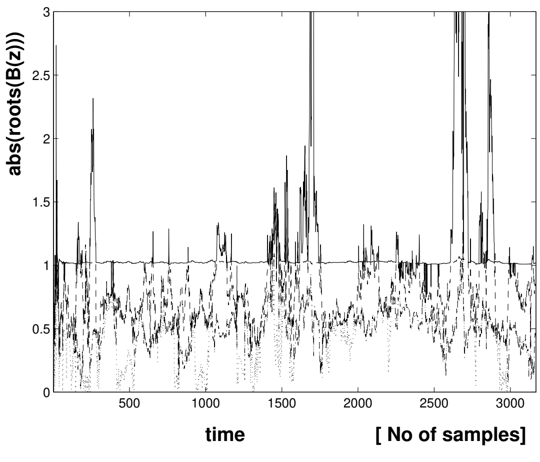


39a. output and setpoint

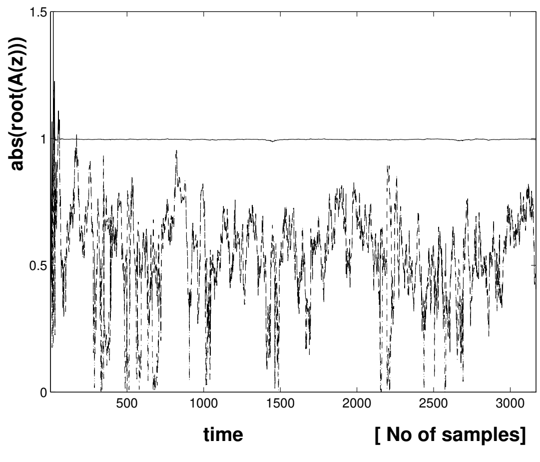


39b. input

Figure 39: Experiment No 17.



40a. roots of the numerator



40b. roots of the denominator

Figure 40: Experiment No 17.

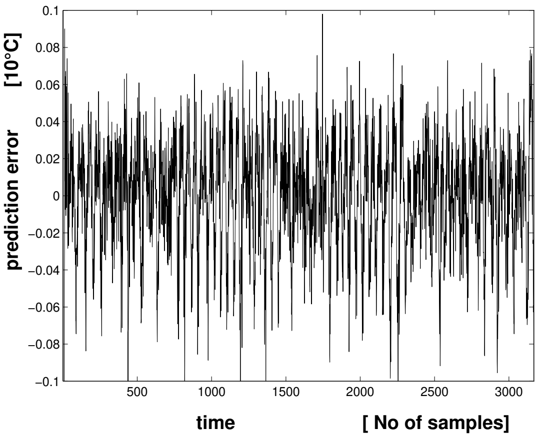
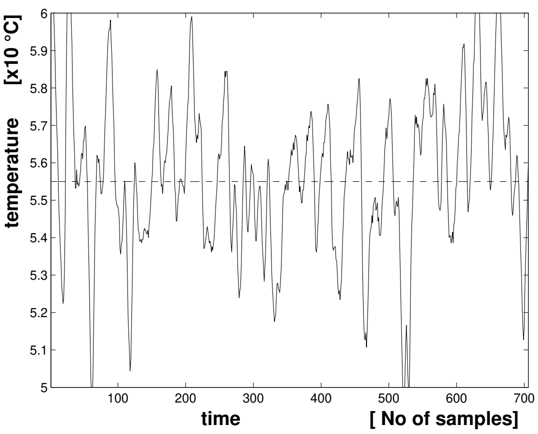
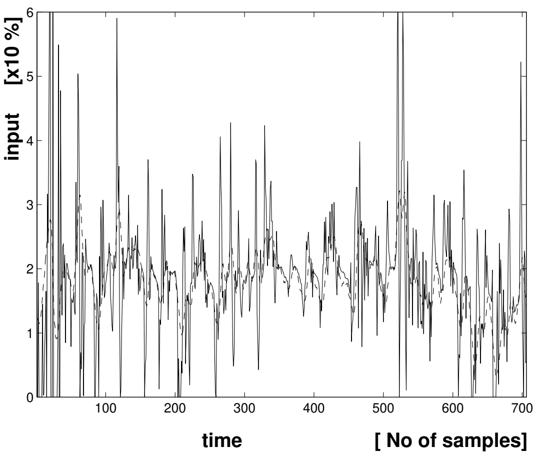


Figure 41: Prediction error in experiment No 17.



42a. output and setpoint



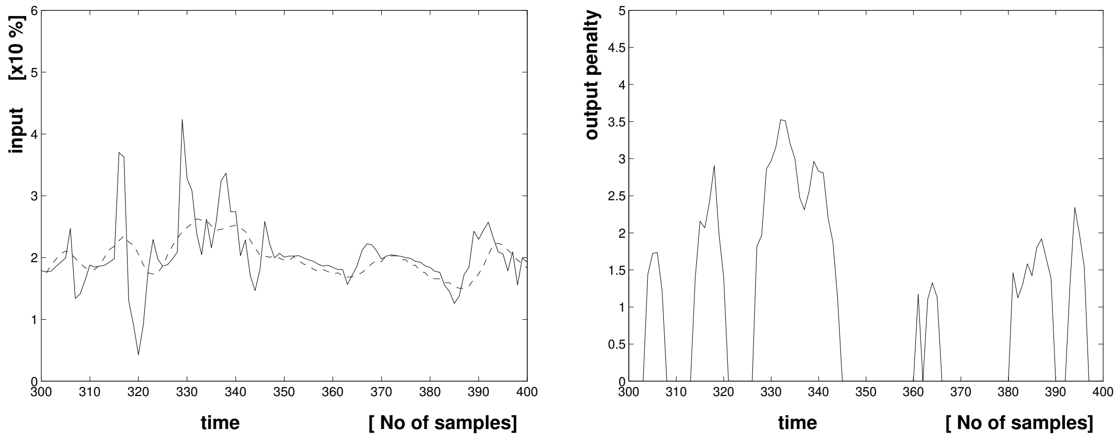
42b. input

Figure 42: Experiment No 18.

now the model was changed to: 3rd order, 2steps of time delay, sampling 10s.

3 Conclusions

Experiments with LQ controllers in the temperature loop of a heat exchanger were performed in three periods with different load and disturbances. In all cases the adaptive LQ control was successful and the input penalization Qu was sufficient knob to tune the behaviour of the controller from the smooth one, when the quality is similar to a PID controller to a “restless” one, however, achieving better quality. The identification results confirmed the second order



43a. Dedail of the input 43b. corresponding penalization of output

Figure 43: Experiment No 18.

model structure with time delay. The choice of time delay was not crucial. The examples with rare disturbances shows how the parameters tend to first order structure and disturbance moves them back to a second order. It can be seen a difference in denominator of the model transfer function obtained in different periods. In all cases the model resulted as nonminimum phase that explains the discrepancy between prediction errors of the model shown and resulting controller behaviour. The experiments have shown that for given purpose a simple PID (in fact practically only P) controller is fully satisfactory for the desired quality. On the other hand verified that the application of an adaptive controller could be also simple and safe. The reachable quality was not much better due to existing delays and nonminimum phase behaviour but it could be expected that in case the disturbances could be measured, the control performance could be improved.

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